1.

a.

$$Y(w) = \frac{1}{2\pi} X(w) * (\pi(\delta(w - w_0) + \delta(w + w_0))) = \frac{1}{2}(X(w - w_0) + X(w + w_0))$$

$Y(w)$ is plotted on figure 1.

b.

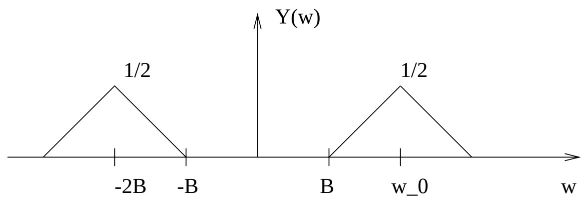

Figure 1: Problem 1: $Y(w)$

Let $v(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$. Then $s(t) = v(t) * \Pi(T/2)$. Therefore,

$$S(w) = V(w)\frac{2sin(Tw/4)}{w} = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(w - \frac{2\pi k}{T})\frac{2sin(Tw/4)}{w} =$$

$$4\pi\sum_{k=-\infty}^{\infty}\delta(w - \frac{2\pi k}{T})\frac{sin(\pi k/2)}{2\pi k} =$$

$$\pi\sum_{k=-\infty}^{\infty}\delta(w - 2Bk)\frac{sin(\pi k/2)}{\pi k/2}$$

$S(w)$ is plotted on figure 2.

c.

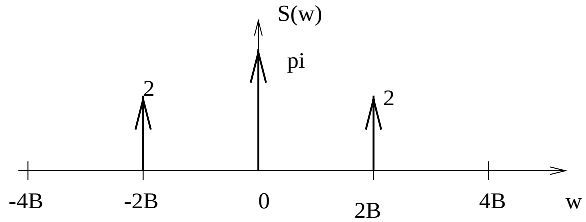

Figure 2: Problem 1: $S(w)$. Note: numbers next to impulses indicate impulse strength.

$$Z(w) = \frac{1}{2\pi} Y(w) * S(w)$$

$Z(w)$ is plotted on figure 3.

d.

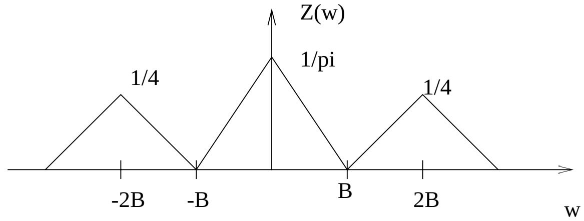

Figure 3: Problem 1: $Z(w)$

From previous part it is obvious that

$$H(w) = \begin{cases} \pi & |w| < B \\ 0 & otherwise \end{cases}$$

2.

a.

Let

$$b_k = \begin{cases} \frac{\pi^2}{3} & k = 0 \\ (-1)^k \frac{4}{k^2} & k > 0 \end{cases}$$

Then $f(t) = \sum_{k=0}^{\infty} b_k cos(kt)$. It follows that

$$F(w) = \pi(\sum_{k=0}^{k=\infty} b_k(\delta(w-k) + \delta(w+k)))$$

Note that

$$g(t) = \frac{1}{\pi^2} f(t/2)\Pi(t/4\pi)$$

, so that

$$G(w) = \frac{1}{\pi^2}\frac{1}{2\pi} F.T.(f(t/2)) * F.T.(\Pi(\frac{t}{4\pi}))$$

$$F.T.(f(t/2)) = 2F(2w) = 2\pi(\sum_{k=0}^{k=\infty} b_k(\delta(2w-k) + \delta(2w+k))) = \pi(\sum_{k=0}^{k=\infty} b_k(\delta(w - \frac{k}{2}) + \delta(w + \frac{k}{2})))$$

where we used $\delta(aw) = \frac{1}{|a|}\delta(w)$ in the last step. Also note that

$$F.T.(\Pi(t/4\pi)) = \frac{2sin(2\pi w)}{w}$$

So that if

$$X(w) = \frac{sin(2\pi w)}{w}$$

and

$$Y(w) = \frac{1}{\pi^2}(\sum_{k=0}^{k=\infty} b_k(\delta(w - \frac{k}{2}) + \delta(w + \frac{k}{2})))$$

Then

$$G(w) = X(w) * Y(w)$$

b.

$$G(w) = X(w) * Y(w) = \frac{1}{\pi^2}(\sum_{k=0}^{k=\infty} b_k(\frac{sin(2\pi(w - \frac{k}{2}))}{w - \frac{k}{2}} + \frac{sin(2\pi(w + \frac{k}{2}))}{w + \frac{k}{2}})) =$$

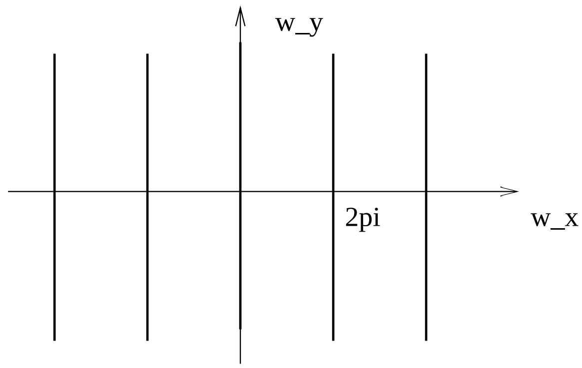

Figure 4: Problem 3: $F_1(w_x, w_y)$. Lines represent impulses in w_x of height 2π.

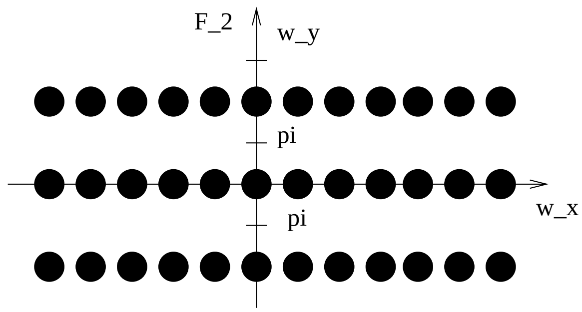

Figure 5: Problem 3: $F_2(w_x, w_y)$. Note: impulse strength is $2\pi^2$.

$$\frac{1}{\pi^2}(\sum_{k=0}^{k=\infty} b_k(\frac{sin(2\pi w - \frac{2\pi k}{2}))}{w - \frac{k}{2}} + \frac{sin(2\pi w + \frac{2\pi k}{2}))}{w + \frac{k}{2}})) =$$

since $sin(x + 2\pi) = sin(x)$ and $sin(x + \pi) = -sin(x)$

$$sin(2\pi w)\frac{1}{\pi^2}\sum_{k=0}^{k=\infty}(-1)^k b_k(\frac{1}{w - \frac{k}{2}} + \frac{1}{w + \frac{k}{2}})$$

3.

NOTE: In this problem, a black circle at position (w_{x0}, w_{y0}) signifies (possibly scaled) $\delta(w_x - w_{x0})\delta(w_y - w_{y0})$.

a.

$$F_1(w_x, w_y) = 2\pi \sum_{k=-\infty}^{\infty} \delta(w_x - 2\pi k)$$

which produces lines of deltas shown in Figure 4.

b.

$$F_3(w_x, w_y) = 2\pi^2 \sum_{k=-\infty}^{\infty} \delta(w_x - \pi k) \sum_{l=-\infty}^{\infty} \delta(w_y - 2\pi l)$$

$F_2(w_x, w_y)$ is shown in Figure 5.

c.

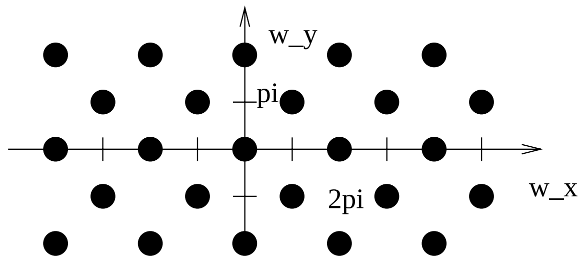

Figure 6: Problem 3: $F_3(w_x, w_y)$. Note: impulses have strength $2\pi^2$.

Let

$$g(x,y) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \delta(x-2k)\delta(y-2l)$$

Then

$$G(w_x, w_y) = \pi^2 \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \delta(w_x - \pi k)\delta(w_y - \pi l)$$

and since $f(x,y) = g(x,y) + g(x+1, y+1)$ we have

$$F_3(w_x, w_y) = G(w_x, w_y) + e^{j(w_x+w_y)} G(w_x, w_y) = \pi^2 (1 + e^{j(w_x+w_y)}) \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \delta(w_x - \pi k)\delta(w_y - \pi l)$$

Note that when $k+l$ is even then $(1 + e^{j(w_x+w_y)}) = 2$, and when $k+l$ is odd then $(1 + e^{j(w_x+w_y)}) = 0$. Thus

$$F_3(w_x, w_y) = 2\pi^2 \sum_{k+l \text{ is even}} \delta(w_x - \pi k)\delta(w_y - \pi l)$$

$F_3(w_x, w_y)$ is shown in Figure 6.

d.

Note that $f_4(x,y) = (f_3(x,y)) * (\Pi(x - 1/2)\Pi(y + 1/2))$. Thus,

$$F_4(w_x, w_y) = F_3(w_x, w_y) e^{-jw_x 1/2} \frac{2sin(w_x/2)}{w_x} e^{jw_y 1/2} \frac{2sin(w_y/2)}{w_y}$$
