University of California at Berkeley
Department of Mechanical Engineering

ME 163
ENGINEERING AERODYNAMICS

FINAL EXAM, 13TH DECEMBER 2005

Answer <u>both</u> questions.
Question 1 is worth 30 marks and question 2 is worth 70 marks.
Time allowed: 3 hours

Open book exam. One text book is allowed. Only hand-written notes are permitted. If you take equations from your written notes, write them down clearly with your working. The parts of the question you should answer are typed in bold. The remainder is explanatory text for the problem. Take care to be consistent with use of units when doing numerical calculations.

Question 1. Supersonic Aerodynamics

i From linearised theory for supersonic aerodynamics the drag coefficient of an aerofoil is

$$C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{4}{\sqrt{M_\infty^2 - 1}} \int_0^1 (\delta_t^2 + \delta_c^2) d\frac{s}{c} \tag{1}$$

where M_∞ is the flight Mach number, α is the aerofoil angle of attack, δ_t is the gradient of the aerofoil thickness profile relative to the chord line, δ_c is the gradient of the aerofoil camber line relative to the chord line, s is the ordinate along the chord line, and c is the aerofoil chord length.

Use this result to show that the minimum drag coefficient for a practical aerofoil section at a given angle of attack is produced by an aerofoil cross section of constant $|\delta_t|$ and zero camber. Sketch what this aerofoil looks like.

Hint: You should use the result that the function $I = \int_0^b g(x, y, y')dx$, where $y' = \frac{dy}{dx}$ and g is an arbitrary function, is minimised when the condition $\frac{\partial g}{\partial y} - \frac{d}{dx}\left(\frac{\partial g}{\partial y'}\right) = 0$ is met.

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ii The Blue Steel missile formed the backbone of the UK strategic nuclear deterrent in the 1950s and 1960s. It was designed to be air-launched from RAF Vulcan and Victor bombers. Following release from the parent aircraft, an Armstrong-Siddeley Stentor rocket motor was fired to boost the missile to a speed of Mach 3 and an altitude of 21.5km (air pressure 0.045 bar), after which it would glide at supersonic speed towards its target. The missile layout is shown in figure 1. Lift is generated by the body, and the fore and aft lifting surfaces. The aft wing is at the same angle of attack as the body, while the foreplane is of variable angle of attack for stability and manouevring. Both fore plane and aft wing sections are uncambered, have negligible thickness and both sets of wings are the same right angled triangular shape with the leading edge sweep angle $\Lambda = 60^o$. The root chord of the foreplane is $1m$, and the root chord of the aft wing is $2.5m$. The missile mass is 7000kg, and the centre of pressure positions of the foreplane and aft wing are indicated on the figure, and you should note that the line of action of the *body only* lift is coincident with the missile c.g. position.

Treating the flows around each part of the missile independently (i.e. no 3-D influence effects) and ignoring all 3-D ef-

fects, starting with trimming calculations (and assume total missile lift=missile weight) determine the missile drag, and hence find the glide slope of the missile in its initial gliding flight phase. Take $\gamma = 1.4$. Use the equations below for the body lift and drag coefficients, and use linearised supersonic aerodynamics for the fore plane and aft wing analysis. Show all your working clearly.

Body lift and drag coefficients referenced to the base area of the body πR^2 are $C_L = 2\alpha$, $C_D = \alpha^2 + 0.005$ for the body angle of attack α in radians.

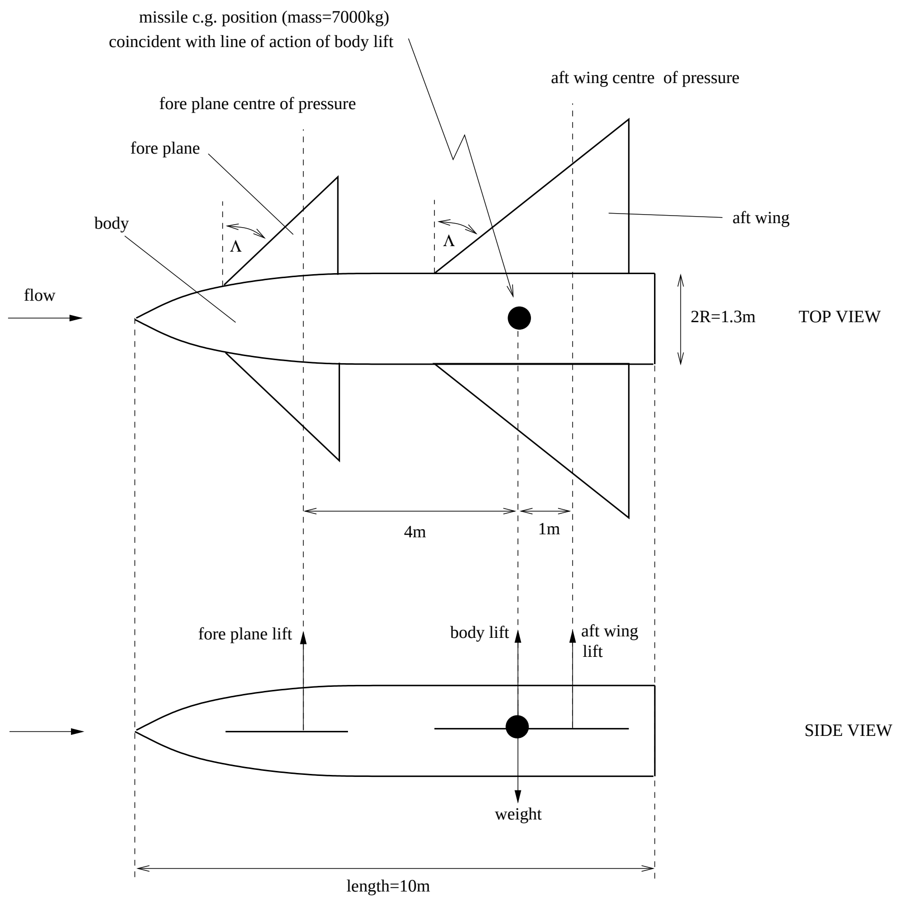

Figure 1: Plan view of the Blue Steel missile showing the missile body, foreplane and aft wing

Question 2. Incompressible Aerodynamics and Flight Mechanics

Consider the design of a low-cost, light aircraft. To give it some useful aerobatic performance it is to have a specified neutral point moment gradient of $\frac{\partial m}{\partial \alpha} = C_{m_\alpha} = -0.4$ (α in radians) and a static margin of $K_n = 0.08$. The wing chord tapers linearly from root to tip, and the wing is untwisted for good inverted flight characteristics. The basic performance and size data are as follows, with all positions measured from the aircraft nose:

Geometric data (full scale)	
Aircraft mass	$m_{a/c} = 550kg$
Aft-most c.g. position along fuselage	$h\,\overline{c} = 1.638m$
Cruise speed	$V_\infty = 60ms^{-1}$
Wing span	$b = 8.08m$
Wing root chord	$c_s = 1.92m$
Wing tip chord	$c_t = 0.772m$
Wing aerodynamic centre position	$h_{n_{wb}}\,\overline{c} = 1.575m$
Tail lift curve slope	$a_t = 0.0658$ per degree
Downwash slope	$\frac{\partial \epsilon}{\partial \alpha} = 0.38$

The same wing section is employed from root to tip, and the aerofoil lift curve slope is $k_o = 2\pi$ per radian.

i **Determine the wing mean aerodynamic chord $\overline{c}$ and aspect ratio.**

ii **For an air density of $\rho = 1.225kgm^{-3}$ calculate the aircraft lift coefficient C_L at the cruise speed.**

iii **Equations 3 and 4 are for the Fourier series representation of an arbitrary wing circulation distribution and the corresponding absolute angle of attack along the span. These equations have been taken from the lecture notes, and the usual notation applies. The first three non-zero Fourier coefficients are to be computed for the above finite wing. The relationship between these Fourier coefficients $\underline{\mathbf{A}}$ and the spanwise absolute angle of attack $\underline{\alpha_\mathbf{a}}$ is given in equation 5. For equal sampling in the θ space verify, to acceptable precision, the values of the coefficients of the third non-zero Fourier coefficient shown in the right hand column of the matrix $\underline{\mathbf{M}}$, with**

the top row of $\underline{\mathbf{M}}$ corresponding to the wing root, and the bottom row to the wing tip.

iv **The inverse of the matrix $\underline{\mathbf{M}}$ is also provided below. Show that the lift curve slope of the wing for the angle of attack in radians is $a_{wing} = 4.741$.**

v **Calculate the wing lift coefficient and induced drag for a wing root absolute angle of attack of 5^o.**

vi **Given the requirements above for C_{m_α} and the static margin, calculate the whole aircraft lift curve slope, assuming that the wing lift curve slope is representative of the wing-body lift curve slope and that the propulsion system has negligible effect upon the longitudinal stability.**

vii **Calculate the required tail area S_t to provide this aircraft lift curve slope.**

viii **Hence calculate the tail volume coefficient V_H and the distance $\bar{l}_t$ between the tail and the wing-body aerodynamic centres.**

$$z = \frac{b}{2} cos\, \theta \tag{2}$$

$$\Gamma = \frac{1}{2} k_{os}\, c_s\, U_\infty \sum_{n=1}^{\infty} A_n\, sin\, n\theta \tag{3}$$

$$\alpha_a(\theta_o) = \frac{k_{os}\, c_s}{k_o(\theta_o) c(\theta_o)} \sum_{n=1}^{\infty} A_n\, sin\, n\theta_o + \frac{k_{os} c_s}{4b} \sum_{n=1}^{\infty} n A_n \frac{sin\, n\theta_o}{sin\theta_o} \tag{4}$$

$$\underline{M}\, \underline{A} = \underline{\alpha}_a \tag{5}$$

$$\underline{\mathbf{M}} = \begin{pmatrix} 1.373 & -2.119 & 2.865 \\ 1.598 & 2.344 & -3.090 \\ 0.373 & 3.357 & 9.325 \end{pmatrix}$$

$$\underline{\mathbf{M}}^{-1} = \begin{pmatrix} 0.3535 & 0.3222 & -0.0018 \\ -0.1760 & 0.1287 & 0.0967 \\ 0.0492 & -0.0592 & 0.0724 \end{pmatrix}$$

Question 1: Solution

(i) Camber contributes to drag but not lift, hence there is no need for camber. An aerofoil must have some thickness for structural strength, so δ_t is non-zero. Do the maths, noting of course that the $\delta_t = y' = \frac{dy}{dx}$ if y is the shape of the thickness profile. Hence by differentiation according to the optimisation function it emerges that $y' = constant$, i.e. the thickness gradient is constant. This means the aerofoil shape is a double-symmetric diamond wedge.

(ii) For the glide slope we need the missile lift to drag ratio. Trim for zero moment about the c.g. and total lift = weight. The lift is the sum of the body lift, foreplane lift and aft plane lift, and use linearised aerodynamics results for the foreplane and aft wing. The body lift coefficient is provided. For the drag use linearised aerodynamics again, ignoring thickness and camber. Hence we need the angles of attack of the foreplane and aft wing. Calculate the wing areas (two triangles each $S_{fore} = 0.577m^2$, $S_{aft} = 3.608m^2$), the base area of the body is $S_b = 1.3273m^2$, and the free stream dynamic head $q_\infty = 28350Nm^{-2}$. The moment about the c.g. gives the foreplane lift in terms of the aft wing lift, hence substitute this into the lift=weight equation. The body and main wing lift coefficients depend on the body angle of attack (AoA α), so we get $\alpha = 0.268rad$. Therefore find the foreplane angle of attack ($0.4187rad$), and hence the drag of each part of the missile. The total drag is $D = 17339N$, so the glide slope is $\epsilon = 14.2^o$.

Question 2: Solution

(i) Linear tapered wing, so $S = 10.88m^2$. From equation for m.a.c. $\overline{c} = 1.43m$.

(ii) Lift=weight, hence aircraft $C_L = 0.225$

(iii) Verify the right hand column of matrix. First three non-zero coefficients are A_1, A_3, A_5. Use θ_o values of $\pi/2$ (wing root), $\pi/4$ ($z = b\sqrt{2}/4$) and 0 (tip). Find chord at the $\pi/4$ location ($1.108m$). 2-D lift curve slope is constant long the span, and the root value is 2π. Note that for $\theta = 0\,,\, \frac{sin(n\theta)}{sin\theta} = n$ Hence the answer!

(iv) For $C_{L_{wing}}$ we need $A_1 = 0.6739\alpha_a$. The wing has no twist to α_a is constant along the span. We find $C_{L_{wing}} = 4.741\alpha_a$, hence the lift curve slope of the wing.

(v) At this angle of attack $C_L = 0.4137$. For C_D evaluate $A_3 = 0.494\alpha_a$ and $A_5 = 0.0624\alpha_a$, so $C_{D_i} = 0.0241$.

(vi) We have specified C_{m_α} and the static margin, so we find the aircraft lift curve slope $C_{L_\alpha} = 5 perradian$. Use the wing lift curve slope as the wing body lift curve slope, and use the relationship between the aircraft lift curve slope and the aircraft wing body and tail lift curve slopes to get the tail area (take care that the tail lift curve slope is expressed per degree), so $S_t = 1.206m^2$. Use the expression for C_{m_α} in terms of c.g. position to get $\overline{V}_H = 0.265$, hence $\overline{l}_t = 3.419m$ and calculate V_H as required.