

## EECS 120, Midterm 1, 3/04/02

Do your calculations on the sheets and put a box around your answer where this makes sense. Print your name and your TA's name here:

Last Name \_\_\_\_\_ First \_\_\_\_\_ TA's name \_\_\_\_\_

1. **20 points** The following statements are either TRUE or FALSE. If you believe a statement is true, outline a BRIEF PROOF. If you believe it is false, provide a BRIEF COUNTEREXAMPLE.

- If  $x(t)$ ,  $t$  is in the set of real numbers, is a real-valued signal, its Fourier transform  $X(f)$ ,  $f$  is in the set of real numbers, is also real-valued.
- If  $x(t)$ ,  $y(t)$ ,  $t$  is in the set of real numbers, are real-valued signals and  $(x * y)(t) = 0$ , for all  $t$  contained in the set of real numbers, then either  $x$  or  $y$  is identically zero.
- If  $x(t)$ ,  $t$  is in the set of real numbers, is a real-valued, baseband signal with bandwidth  $W$  Hz, then the signal  $y$ ,  $y(t) = x^4(t)$ ,  $t$  is in the set of real numbers, has bandwidth at most  $4W$  Hz.
- If  $x(t)$ ,  $t$  is in the set of real numbers, is a real-valued, band-limited signal with bandwidth  $W$  Hz, then the signal  $y(t) = x(2t)$ ,  $t$  is in the set of real numbers, has bandwidth  $W^2$  Hz.
- If  $x, y$  are real-valued signals with bandwidth  $W_x, W_y$  Hz, respectively, the signal  $x + y$  has bandwidth  $W_x + W_y$  Hz.

2. **20 points**  $m_1, m_2$  are two signals both with bandwidth  $B$  Hz. A modulated signal  $x$  with carrier frequency  $f_c \gg B$  is constructed as

$$\text{For every value } t, x(t) = m_1(t)\cos 2\pi f_c t + m_2(t)\sin 2\pi f_c t$$

- Find a coherent demodulation scheme that recovers  $m_1$ . Briefly explain using a mathematical or graphical argument why your scheme works.
- Find a coherent demodulation scheme that recovers  $m_2$ . Briefly explain using a mathematical or graphical argument why your scheme works.

3. **20 points** A pure tone  $m(t) = \cos 2\pi f_m t$  amplitude-modulates the carrier  $\cos 2\pi f_c t$  ( $f_c \gg f_m$ ) using three schemes: (1) AM without carrier, (2) AM with large carrier, (3) AM-USB. The resulting signal is called  $x$ .

For each scheme write down the algebraic expression for  $x$ , the algebraic expression for its Fourier transform,  $X$ , and sketch  $X(f)$ , for  $f$  greater than or equal to zero. Carefully mark the magnitudes and frequencies on your sketch.

4. **20 points** A signal  $m$  phase-modulates a carrier of frequency  $f_c$  Hz to produce the signal

$$\text{For every value } t, x(t) = \cos(2\pi f_c t + m(t))$$

Suppose  $|m(t)| \ll 1$ , so this is narrow-band PM.

- a. Find a coherent demodulation scheme to recover the signal  $m$  from  $x$ . Explain why your scheme works. You may give an algebraic or block diagram description of your scheme.
  
- b. Suppose the modulated signal suffers amplitude distortion so that the received signal is  $y$  instead of  $x$ ,

$$\text{For every value } t, y(t) = A(t)x(t) = A(t)\cos(2\pi f_c t + m(t))$$

Where  $1 \leq A(t) \leq 2$  is the distortion. What signal does your demodulator generate and how is it related to  $m$ ?

- c. Modify the design of your demodulator so that the effect of the distortion  $A$  is eliminated. Remember you don't know  $A$ . [Hint: First send  $y$  through a hard limiter. A hard limiter is a memoryless device  $g$  whose output is  $\text{sgn}(y(t))$  when its input is  $y(t)$ .]