EECS 120/Fall 1990
Final
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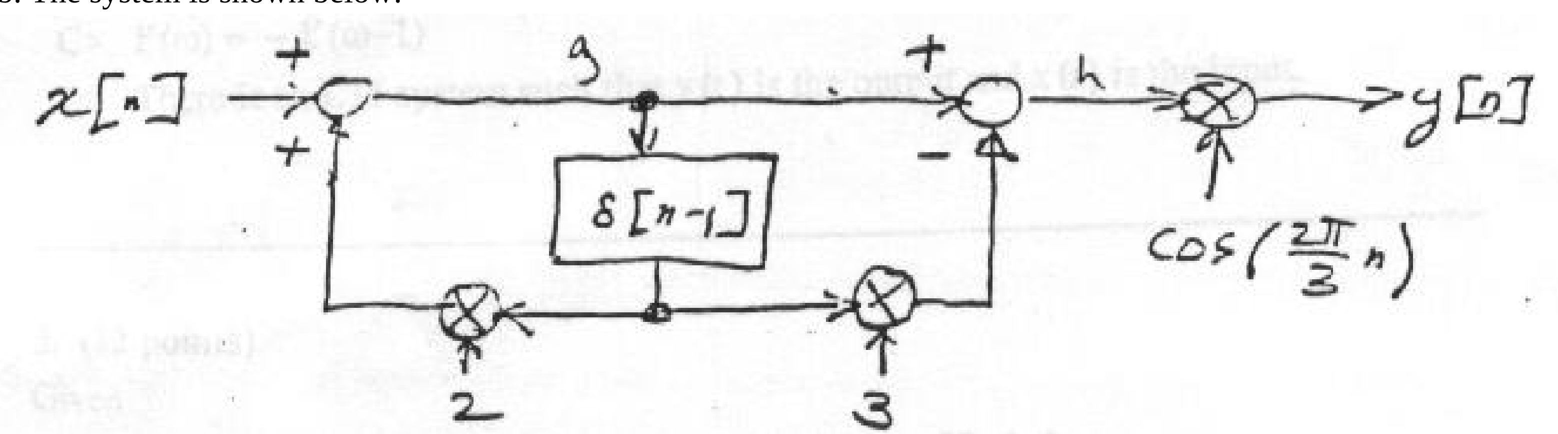
This examination is **closed book** and **closed notes**. You are, however, allowed four pages of notes and formulas, both sides.
Answer all the questions in a blue book. Loose sheets of paper are not allowed in the exam except by special permission of the exam proctor.
There are three questions, for a total possible score of 100 points.
If you don't understand a question, please ask the exam proctor for clarification.

Problem #1 (8 points)

For each of the systems below, mark true or false on the accompanying answer sheet in each blank provided, and be sure and include this answer sheet in your blue book!
a. y(t) = x(t){4 + x(t - 3)}
b. The system is shown below:



c. y(t) = sin((**omega sub nought**)t + Kx(t))
d. y(t) = x(t)***delta**(t-4) + x(t-f)sinc(t-2)

Problem #2

a. x(t) is a real-valued and periodic with Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k}$$

Also define

$$y(t) = \sum_{k=-\infty}^{\infty} a_k^{*} e^{j\omega_0 K}$$

Which of the following are true statements (circle the true statements on the attached answer sheet)?:
A> y(t) = x*(t)
B> y(t) = x(-t)
C> y(t) is a complex-valued signal.
D> y(t) is not periodic.

b. x(t) is real-valued and y(t) = - x(t-1). Which of the following are true statements (circle the true statements on the attached answer sheet)?: A> The amplitude spectrum of y(t) is the same as the amplitude spectrum of x(t).
B> The phase spectrum of y(t) is the negative of the phase spectrum of x(t).
C> Y(w) = -X(**omega**-1)
D> There is an LTI system such that y(t) is the output and x(t) is the input.

Problem #3

Given $x[n] = \alpha^n u[-n], \qquad h[n] = \beta^n u[n]$

where **alpha** and **beta** are real-valued parameters.
a. State conditions on alpha and beta such that the Fourier transform of x[n] and h[n] *both* exist.
b. For the conditions of a., find x[n]*h[n] using the definition of convolution in the time domain.
c. Repeat b. using Fourier transform techniques. d. State conditions on **alpha** and **beta** such that the Z-transform of the signal {x[n]+h[n]} exists. Give the region of convergence for the Z-transform under these conditions.

Problem #4

Given the signals

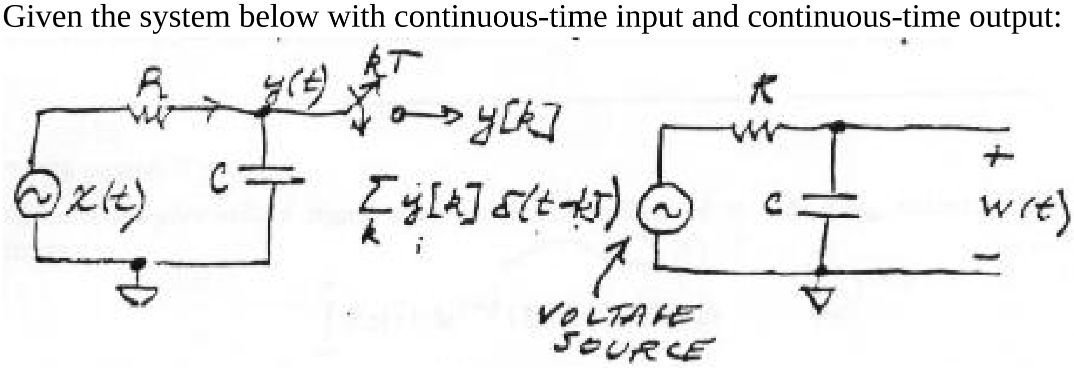
$$h(t) = e^{-t} \sin(2\pi t / T_1) u(t)$$

$$x(t) = \sum_{k=-\infty}^{\infty} h(t - kT_0)$$

a. Find the Fourier transform H(**omega**) of h(t).
b. Sketch qualitatively what the Fourier transform X(**omega**) of x(t) should look like.
c. Derive a mathematical expression for the Fourier transform X(**omega**) of x(t).

Problem #5

Given the system below with continuous-time input and continuous-time output:



a. Find the transfer funciton of the continuous-time system from input x(t) to output y(t).
b. State conditions under which x(t), -infinity < t < infinity could in principle be recovered from w(t), -infinity < t < infinity; that is the system is invertible.
c. State conditions under which the continuous-time system with input x(t) and output w(t) is LTI, or if it is never LTI so stae. Give reasons for your answer.
d. For the conditions of c, find the transfer function of the system from input x(t) to output w(t). If the system is never LTI, find the Fourier transform W(**omega**) of w(t) in terms of the Fourier transform X(**omega**) of x(t).

Problem #6

Given a continous-time signal x(t), define a discrete-time signal

$$x[k] = \int_{kT}^{(k+1)T} x(t)\,dt\ .$$

a. State some conditions, the most general you can think of, under which x(t), -infinity < t < infinity can be recovered from x[k], -infinity < k < infinity.
b. Under the conditions of a., show how you would actually recover x(t) from x[k].
c. Repeat a. for

$$x[k] = \int_{-\infty}^{\infty} g(t-kT)x(t)\,dt$$

for some signal g(t) with Fourier transform G(**omega**).
d. Repeat for b. for the signal of part c.

Problem #7

Given a *complex-valued* signal z(t) which is bandlimited to |**omega**| < **omega sub nought**, evaluate the integral

$$\int_{-\infty}^{\infty} \mathrm{Re}\{z(t)e^{j\omega_0 t}\}\,\mathrm{Im}\{z(t)e^{j\omega_0 t}\}\,dt$$

in terms of **omega sub nought** and Z(**omega**), the Fourier transform of z(t). HINT: Be sure and utilize the bandlimited information!

Problem #8

Given a continuous-time signal x(t), and a discrete-time signal a[n] that is *periodic* with period N,

a[n+N] = a[n].

Form the discrete-time signal

y[n] = a[n]x(nT).

a. Find the discrete-time Fourier transform Y(**ohm**) of y[n]. Sketch what it looks like.
b. State conditions on x(t) or X(**omega**), the most general you can think of, under which x(t), -infinity < t < infinity can be recovered from y[n], -infinity < n < inifinity, for any {a[0], ... , a[N-1]}.
