ME 132, Fall 2003, Quiz 1

Name:

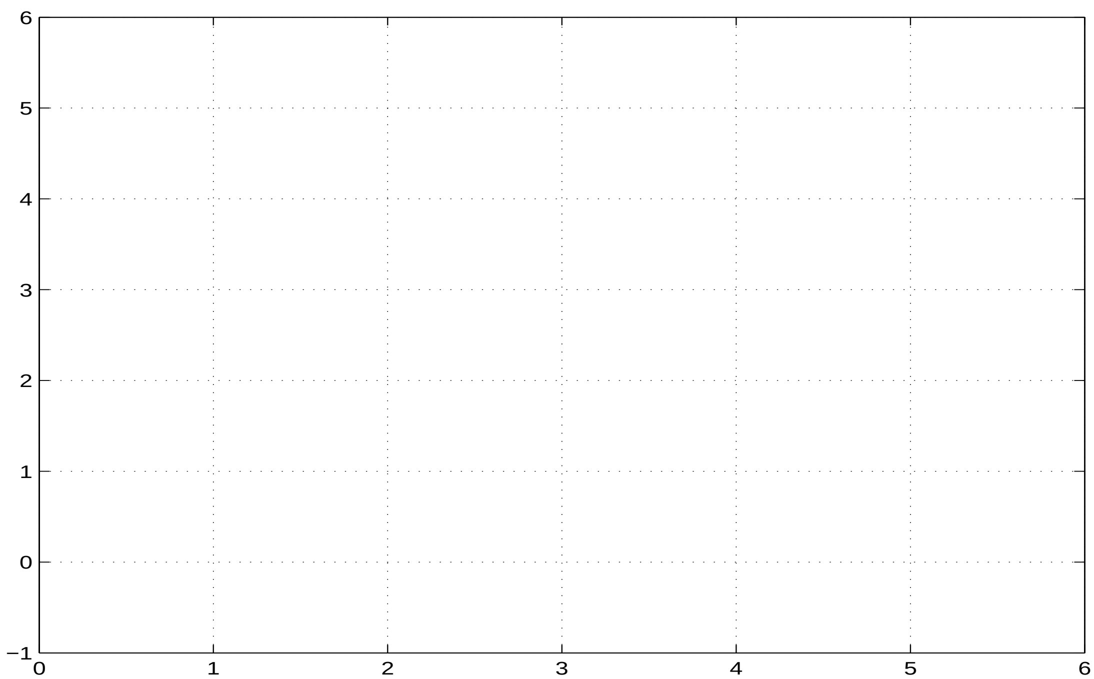
# 1	# 2	# 3	# 4	# 5	TOTAL
8	8	12	9	13	50

1. Neatly/accurately sketch the solution (for $t \geq 0$) to the differential equation

$$\dot{x}(t) = -3x(t) + 6u(t)$$

subject to the initial condition $x(0) = -1$, and forcing function

$$\begin{array}{ll} u(t) = 0 & \text{for } 0 \leq t \leq 2 \\ u(t) = 3 & \text{for } 2 < t \leq 4 \\ u(t) = 2 & \text{for } 4 < t \leq 4.33 \\ u(t) = 1 & \text{for } 4.33 < t \leq 6 \end{array}$$



2. (a) What is the general form of the solution to the differential equation

$$\ddot{x}(t) + 6\dot{x}(t) + 5x(t) = 0$$

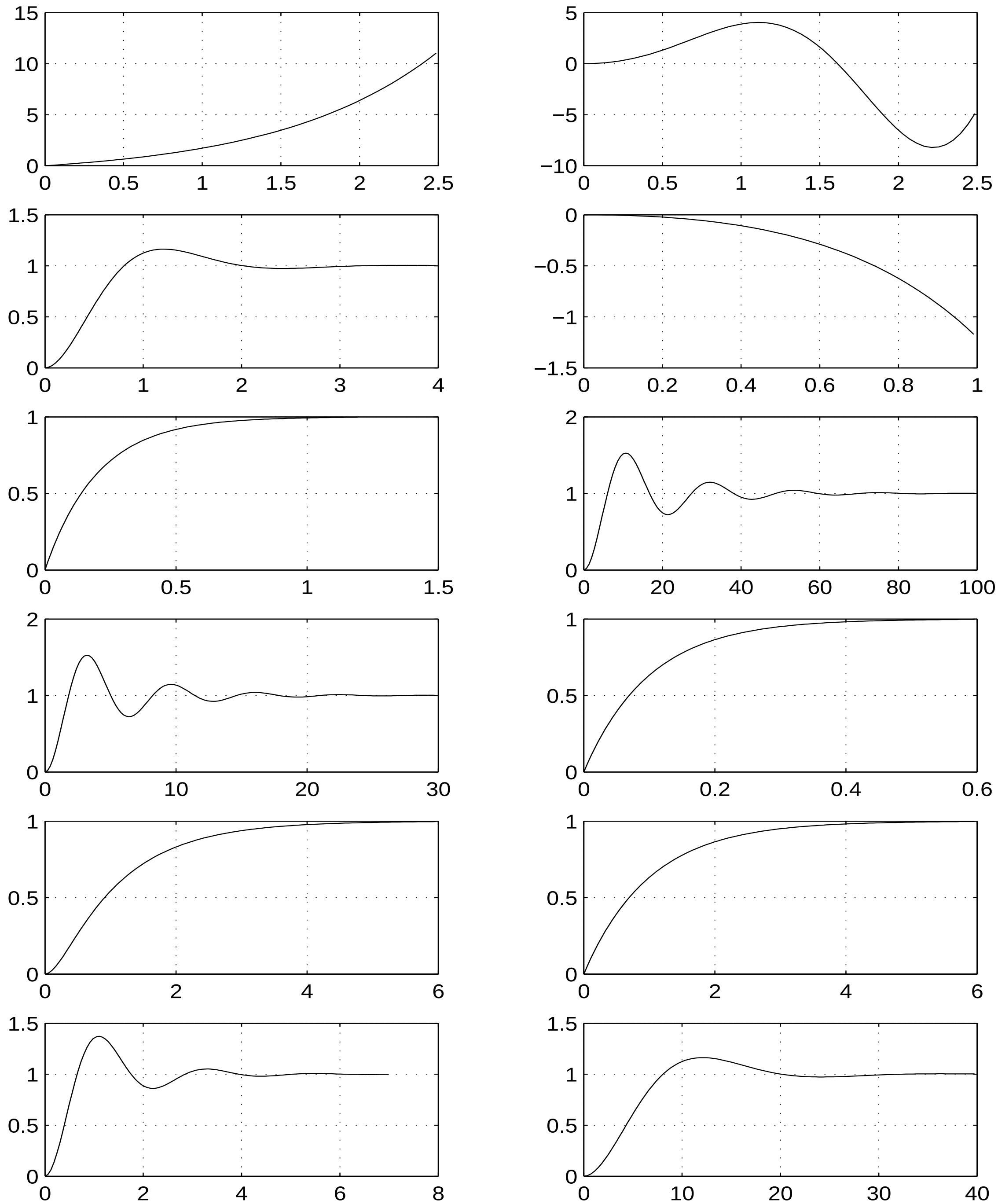
(b) What is the general form of the solution to the differential equation

$$\ddot{x}(t) + 6\dot{x}(t) + 5x(t) = -10$$

Your expressions both should have two free constants.

3. The response of the systems listed below is shown on the next page. Match the ODE with the solution graph. Make a table with pertinent information that justifies your answers. In each case, all appropriate initial conditions are 0. By that, I mean that if the system is first-order, then the initial condition is $y(0) = 0$. If the system is second order, then the initial conditions are $\dot{y}(0) = 0, y(0) = 0$. And so on...

   (a) $\dot{y}(t) + y(t) = 1$
   (b) $\dot{y}(t) + 5y(t) = 5$
   (c) $\dot{y}(t) - y(t) = 1$
   (d) $\dot{y}(t) + 10y(t) = 10$
   (e) $\ddot{y}(t) - 2\dot{y}(t) - y(t) = -1$
   (f) $\ddot{y}(t) - 2\dot{y}(t) + 9y(t) = 9$
   (g) $\ddot{y}(t) + 0.4\dot{y}(t) + y(t) = 1$
   (h) $\ddot{y}(t) + 0.12\dot{y}(t) + 0.09y(t) = 0.09$
   (i) $\ddot{y}(t) + 6\dot{y}(t) + 5y(t) = 5$
   (j) $\ddot{y}(t) + 0.3\dot{y}(t) + 0.09y(t) = 0.09$
   (k) $\ddot{y}(t) + 3\dot{y}(t) + 9y(t) = 9$
   (l) $\ddot{y}(t) + 1.8\dot{y}(t) + 9y(t) = 9$


15 10 5 0
0 0.5 1 1.5 2 2.5
5 0 −5 −10
0 0.5 1 1.5 2 2.5
1.5 1 0.5 0
0 1 2 3 4
0 −0.5 −1 −1.5
0 0.2 0.4 0.6 0.8 1
1 0.5 0
0 0.5 1 1.5
2 1 0
0 20 40 60 80 100
2 1 0
0 10 20 30
1 0.5 0
0 0.2 0.4 0.6
1 0.5 0
0 2 4 6
1 0.5 0
0 2 4 6
1.5 1 0.5 0
0 2 4 6 8
1.5 1 0.5 0
0 10 20 30 40

4. A process, with input u, and output y, is governed by the equation

$$\ddot{y}(t) + \dot{y}(t) + y(t) = u(t)$$

A PI (Proportional plus Integral) controller is proposed

$$\begin{aligned} u(t) &= K_P\left[r(t) - y(t)\right] + K_I z(t) \\ \dot{z}(t) &= r(t) - y(t) \end{aligned}$$

Here r is a reference input.

(a) Using the controller equations, express $\dot{u}(t)$ in terms of $r, \dot{r}, y$ and $\dot{y}$.

(b) By differentiating the process equation, and substituting, derive the closed-loop <u>differential</u> equation relating r and y (there should be no u in the equation).

(c) Using the 3rd order test for stability, determine the conditions on K_P and K_I such that the closed-loop system is stable.

5. A process, with input u, disturbance d and output y is governed by

$$\dot{y}(t) = y(t) + 3u(t) + d(t)$$

(a) Is the process stable?

(b) Suppose $y(0) = 2$, and $u(t) = d(t) \equiv 0$ for all $t \geq 0$. What is the solution $y(t)$ for $t \geq 0$.

(c) Consider a proportional-control strategy, $u(t) = K_1 r(t) + K_2 [r(t) - y(t)]$. Determine the closed-loop differential equation relating the variables (y, r, d).

(d) For what values of K_1 and K_2 is the closed-loop system stable?

(e) As a function of K_2, what is the steady-state gain (just at zero frequency) from $d \rightarrow y$ in the closed-loop system?

(f) As a function of K_1 and K_2, what is the steady-state gain (again, just at zero frequency) from $r \rightarrow y$ in the closed-loop system?

(g) Choose K_1 and K_2 so that the steady-state gain from $r \rightarrow y$ equals 1, and the steady-state gain from $d \rightarrow y$ equals 0.2.

(h) With those gains chosen, sketch (try to be accurate) the two responses $y(t)$ and $u(t)$ for the following situation:

$$y(0) = 0, \quad r(t) = \begin{cases} 0 & \text{for } 0 \le t \le 1 \\ 1 & \text{for } 1 < t \end{cases}, \quad d(t) = \begin{cases} 0 & \text{for } 0 \le t \le 2 \\ 1 & \text{for } 2 < t \end{cases}$$