EECS 40, Spring 2005
Prof. Chang-Hasnain
Midterm #2

April 14, 2005
Total Time Allotted: 80 minutes

1. This is a closed book exam. However, you are allowed to bring one page (8.5" x 11"), double-sided notes
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. Numerical answers within a factor of 1.5 will not get points deducted, provided the steps are all correct and the errors are due to the lack of a calculator. (e.g. if the correct answer is 1, the acceptable range will be 0.67~1.5).
4. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
5. Write your answers in the spaces (lines, boxes or plots) provided.
6. Remember to put down units. Points will be taken off for answers without units.
7. Mobility chart is provided for your reference.

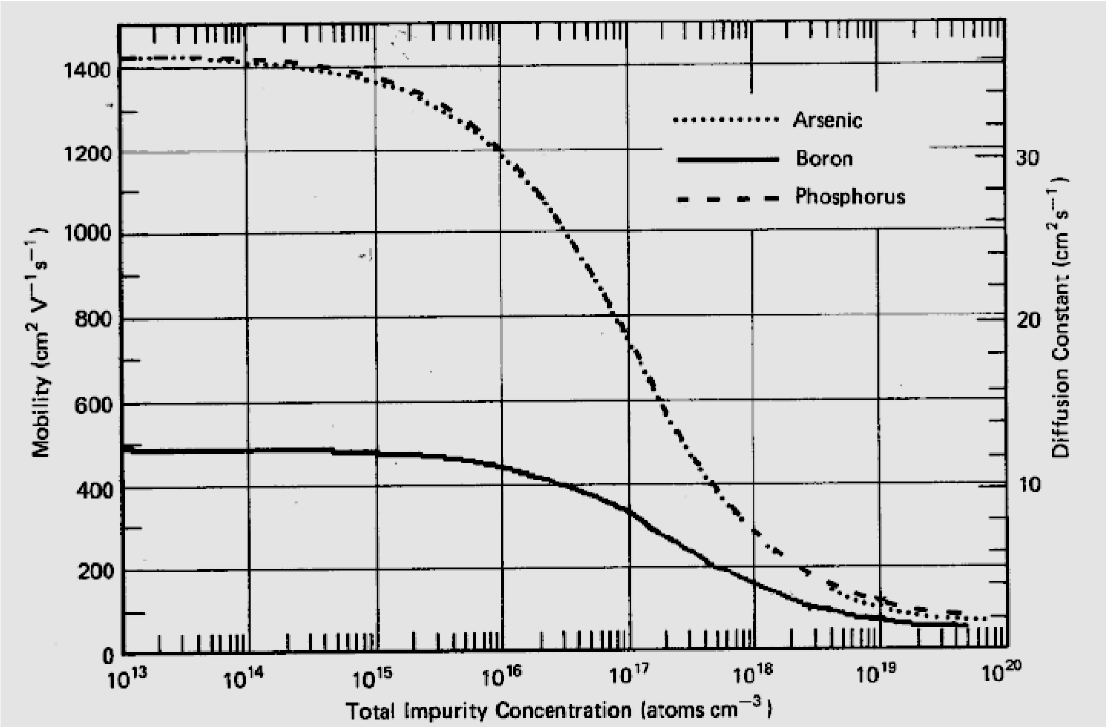
Last (Family) Name:______________________________

First Name: ______________________________

Student ID: ______________________________

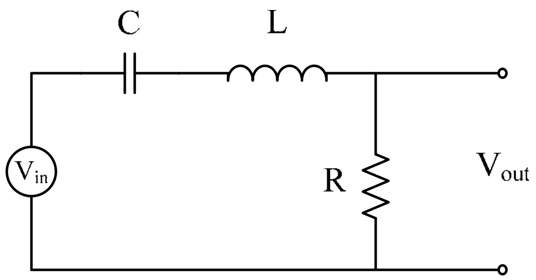
Signature: ______________________________

Score:	
Problem 1 (25 pts)	
Problem 2 (25 pts):	
Problem 3 (20 pts):	
Problem 4 (30 pts)	
Total 100 pts	



1400
1200
1000
800
600
400
200
0
Mobility (cm^2 V^{-1}s^{-1})
Arsenic
Boron
Phosphorus
30
20
10
Diffusion Constant (cm^{2}s^{-1})
10^{13}
10^{14}
10^{15}
10^{16}
10^{17}
10^{18}
10^{19}
10^{20}
Total Impurity Concentration (atoms cm^{-3})

1. [25 pts] RLC circuit in series. Given: $R = 1 + \frac{1}{\sqrt{10}}\,\Omega$, $C = 1$ F, and $L = \frac{1}{\sqrt{10}}$ H.



Part a. This is a simple voltage divider:

$$\mathbf{V}_{out} = \mathbf{V}_{in} \frac{\mathbf{Z}_R}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C}$$

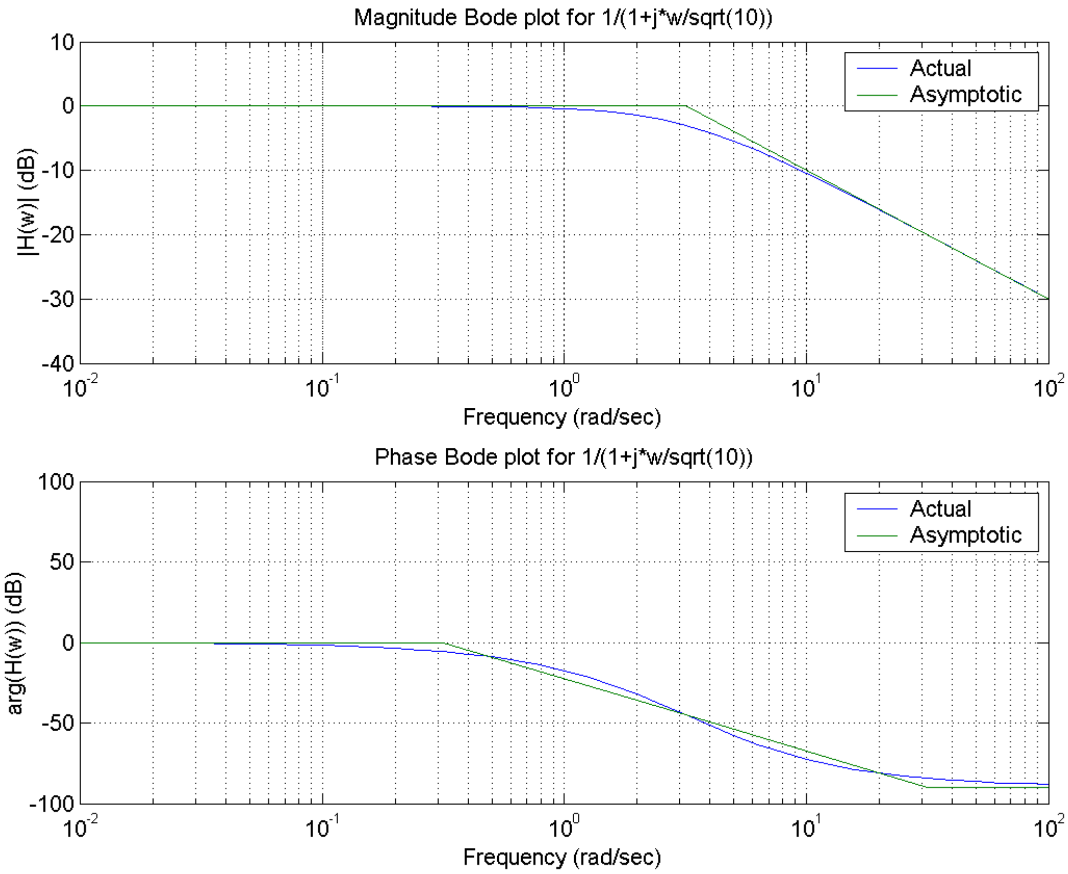
where

$$\mathbf{Z}_R = R$$
$$\mathbf{Z}_L = j\omega L$$
$$\mathbf{Z}_C = \frac{1}{j\omega C}$$

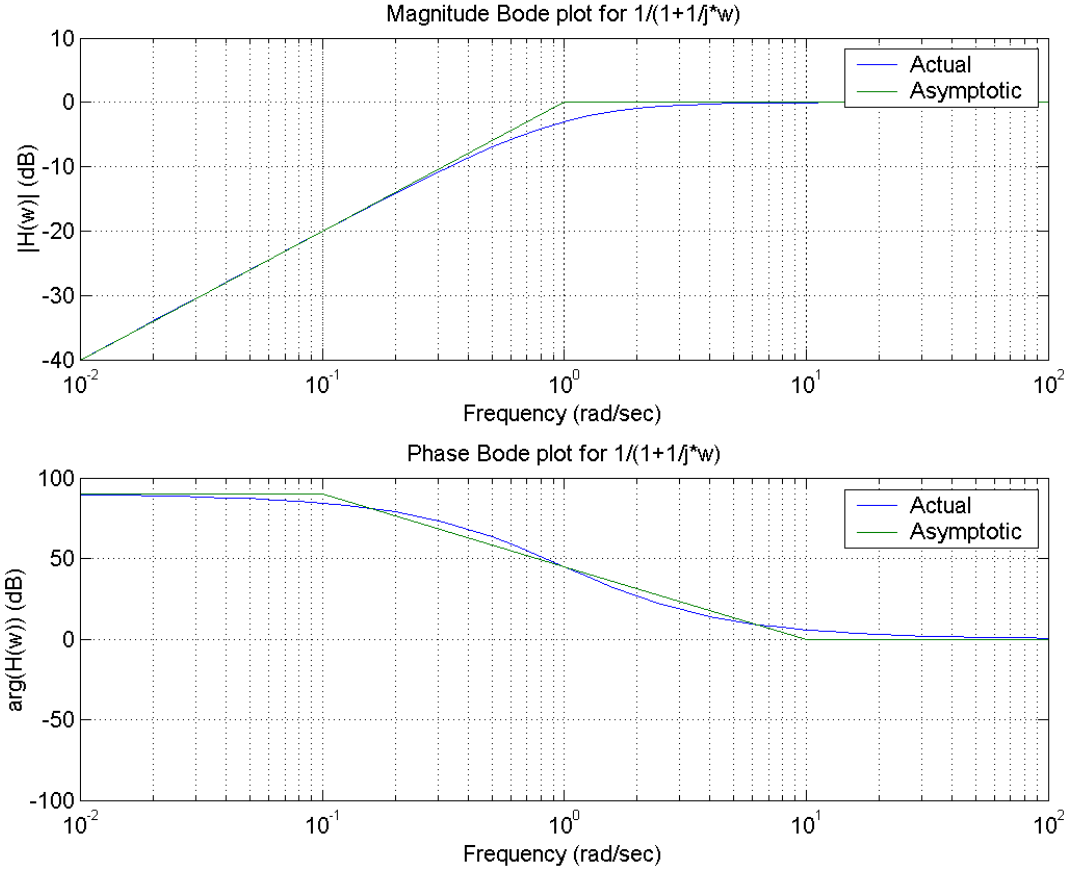
$$\begin{aligned} H(\omega) &= \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} \\ &= \frac{R}{R + j\omega L + \frac{1}{j\omega C}} \\ &= \frac{1 + \frac{1}{\sqrt{10}}}{\frac{j\omega}{\sqrt{10}} + \left(1 + \frac{1}{\sqrt{10}}\right) + \frac{1}{j\omega}} \\ &= \boxed{\frac{1 + \frac{1}{\sqrt{10}}}{\left(\frac{j\omega}{\sqrt{10}} + 1\right)\left(1 + \frac{1}{j\omega}\right)}} \end{aligned}$$

Part b. We can analyze this transfer by breaking it down into first order parts.

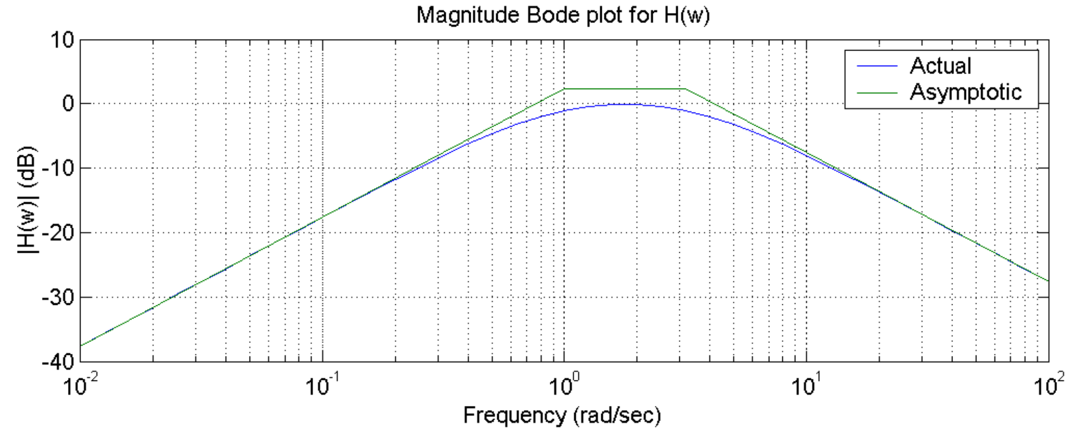
$$\frac{1}{1 + \frac{j\omega}{\sqrt{10}}}:$$

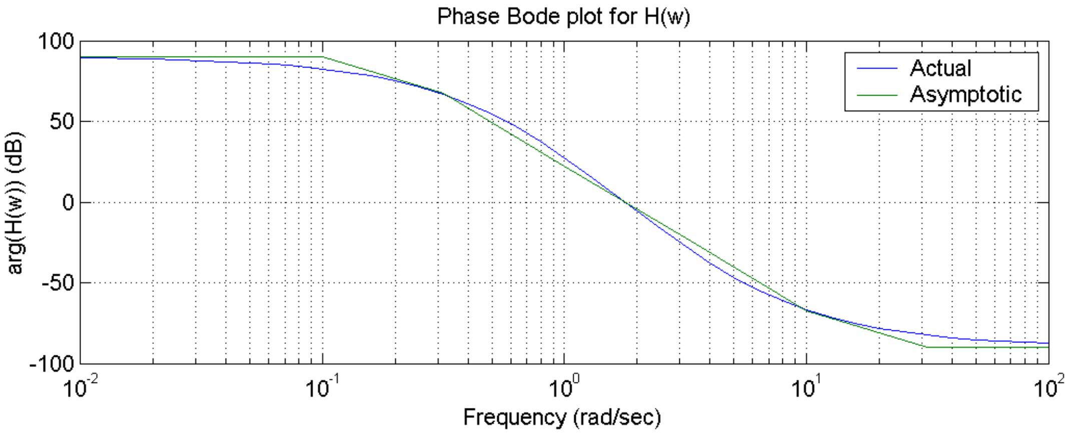

Magnitude Bode plot for 1/(1+j*w/sqrt(10))
10
0
-10
-20
-30
-40
|H(w)| (dB)
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)
Actual
Asymptotic
Phase Bode plot for 1/(1+j*w/sqrt(10))
100
50
0
-50
-100
arg(H(w)) (dB)
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)
Actual
Asymptotic

$$\frac{1}{1+\frac{1}{j\omega}}:$$



$H(\omega)$: We are given that $\log_{10}\left(1+\frac{1}{\sqrt{10}}\right)\approx 0.12$, thus $20\log_{10}\left(1+\frac{1}{\sqrt{10}}\right)\approx 2.4$ dB, which actually is accurate out to the 2nd significant digit. So we can superposition the above two plots and shift the magnitude plot up by 2.4 dB.





Part c. By superposition, we can treat each frequency separately. For $\omega = 1$ rad/sec:

$$H(1) = \frac{1 + \frac{1}{\sqrt{10}}}{\left(1 + \frac{j}{\sqrt{10}}\right)\left(1 + \frac{1}{j}\right)} \approx \frac{1.3\angle 0^\circ}{(1\angle 18^\circ)(1.4\angle -45^\circ)}$$

$$|H(1)| \approx \frac{1.3}{1.4} \approx 0.9$$

$$\angle H(1) \approx 0^\circ - 18^\circ - (-45^\circ) = 27^\circ$$

For $\omega = 10$ rad/sec:

$$H(1) = \frac{1 + \frac{1}{\sqrt{10}}}{\left(1 + j\sqrt{10}\right)\left(1 + \frac{1}{j10}\right)} \approx \frac{1.3\angle 0^\circ}{\left(3.3\angle 72^\bullet\right)\left(1\angle - 6^\circ\right)}$$

$$\left|H(1)\right| = \frac{1.3}{3.3} = 0.4$$

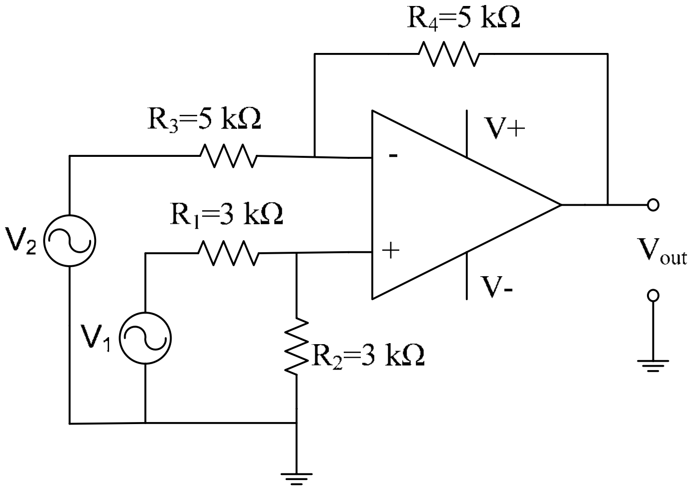
$$\angle H(1) = 0^\circ - 72^\circ - \left(-6^\circ\right) = -66^\circ$$

$$\begin{aligned} \mathbf{V}_{out} &= H(1) \cdot 2\cos t + H(10) \cdot 3\sin 10t \\ &= \left(0.9\angle 27^\circ\right) \cdot 2\cos t + \left(0.4\angle - 66^\circ\right) \cdot 3\cos\left(10t - 90^\circ\right) \\ &= 1.8\cos\left(t + 27^\circ\right) + 1.2\cos\left(10t - 156^\circ\right) \end{aligned}$$

The actual answer is:

$$\mathbf{V}_{out} = \boxed{1.775\cos\left(t + 27.45°\right) + 1.185\cos\left(10t - 156.7°\right)}$$

2. [25 pts] The op-amp is configured as shown in the figure.



(a) Express V_{out} in terms of V_1 and V_2. [5 pts]

Use superposition.
Short V_2 first.

Then we have $V_+ = \frac{3}{3+3}V_1 = \frac{1}{2}V_1$, then $V_- = \frac{5}{5+5}V_{out} = \frac{1}{2}V_{out} = V_+ = \frac{1}{2}V_1$

So we have $V_{out} = V_1$

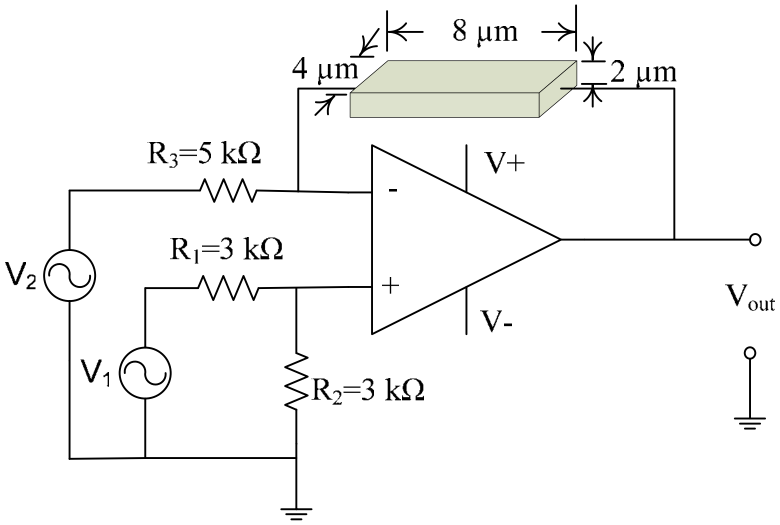
Then we short V_1.

We have $V_+ = V_- = 0$ and $\frac{V_2}{5k} = -\frac{V_{out}}{5k} \Rightarrow V_{out} = -V_2$

Now we superpose the above two results:

$$V_{out} = V_1 - V_2$$

(b) Now resistor R_4 is changed to a piece of Silicon whose dimensions are L=8 μm, T=2 μm, and W=4 μm as labeled in the figure. The Silicon is doped with Boron at a concentration of 3×10^{16} cm^{-3}. Express V_{out} in terms of V_1 and V_2 again. [10 pts]



We need to find the resistance of the Silicon first.

Resistivity is: $\rho = \dfrac{1}{qn\mu_n + qp\mu_p}$

Since boron is acceptors, we have $p \cong N_A = 3\times10^{16} cm^{-3}$ and $n = \dfrac{n_i^2}{p} \cong 3\times10^{3} cm^{-3}$.

Since the electron density is much smaller than the hole density, we approximate resistivity as:

$$\rho = \frac{1}{qp\mu_p}$$

Since $N_A + N_D = 3\times10^{16} cm^{-3}$, from the mobility chart, we get $\mu_p = 420 cm^2 / V / s$.

So $\rho = \dfrac{1}{1.6\times10^{-19}\times3\times10^{16}\times420} = \dfrac{1}{2.016} \cong 0.5\Omega cm$

Resistance is: $R = \rho\dfrac{L}{WT} = 0.5\dfrac{8}{4\times2\times10^{-4}} = 5\,\text{k}\Omega$

So the same as (a),

$$V_{out} = V_1 - V_2$$

(c) If we add 5×10^{16} cm^{-3} Phosphorus into the Silicon, what is V_{out} now? [10 pts]

Since phosphorus is a donor, now we have $N_A = 3\times10^{16} cm^{-3}$ and $N_D = 5\times10^{16} cm^{-3}$.

$N_D - N_A = 2\times10^{16} cm^{-3} >> 0$, so $N_D >> N_A$, the material is N-type now. And we have

$n = N_D - N_A = 2\times10^{16} cm^{-3}$, $p = \frac{n_i^2}{n} = 5\times10^3 cm^{-3}$.

Now resistivity can be approximated as $\rho = \frac{1}{qn\mu_n}$

Since $N_A + N_D = 8\times10^{16} cm^{-3}$, from the mobility chart, we get $\mu_n = 800 cm^2/V/s$

So $\rho = \frac{1}{1.6\times10^{-19}\times2\times10^{16}\times800} = \frac{1}{2.56} \cong 0.4\Omega cm$

Resistance is: $R = \rho\frac{L}{WT} = 0.4\frac{8}{4\times2\times10^{-4}} = 4\ \text{k}\Omega$

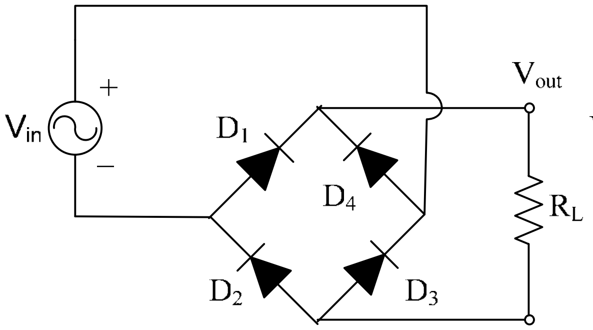
So for V_1, we have $V_- = \frac{5}{5+4}V_{out} = \frac{1}{2}V_1$, so $V_{out} = 0.9V_1$

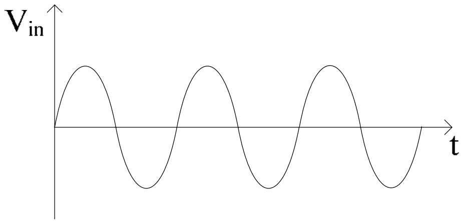
For V_2, we have $\frac{V_2}{5k} = -\frac{V_{out}}{4k}$, so $V_{out} = -0.8V_2$

Finally,

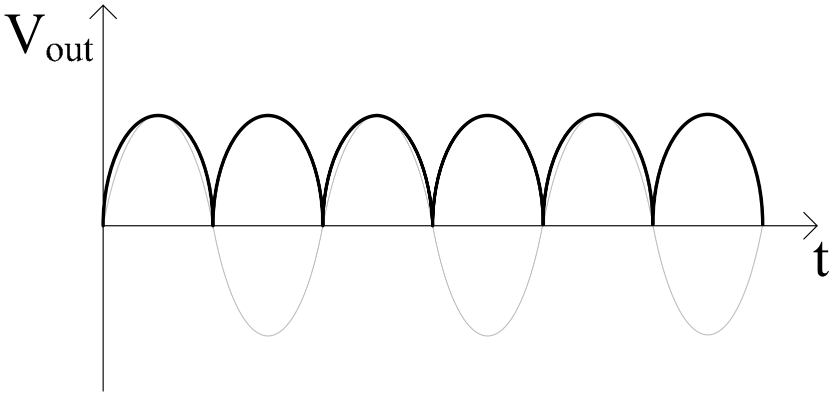
$$V_{out} = 0.9V_1 - 0.8V_2$$

3. [25 pts] Diode rectifier. The input voltage is shown below. Assume all the diodes are ideal and with threshold voltage V_T=0V.

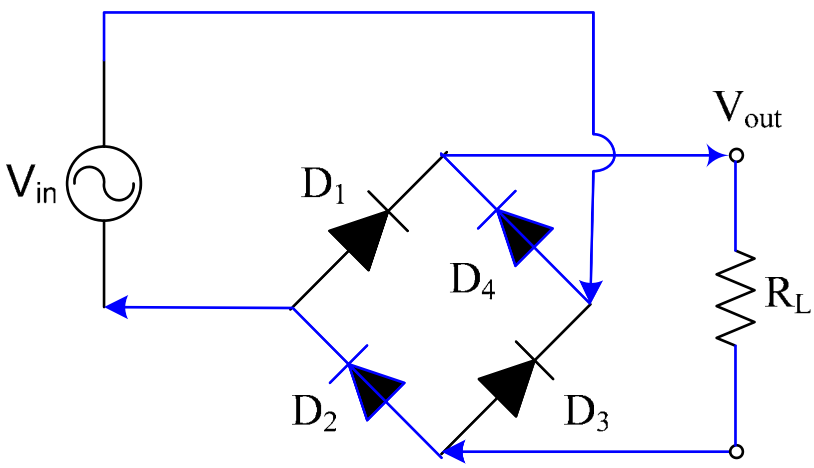




(a) Please draw the output voltage on the load resistor and give explanations. (The input is given as a reference.) [8 pts]

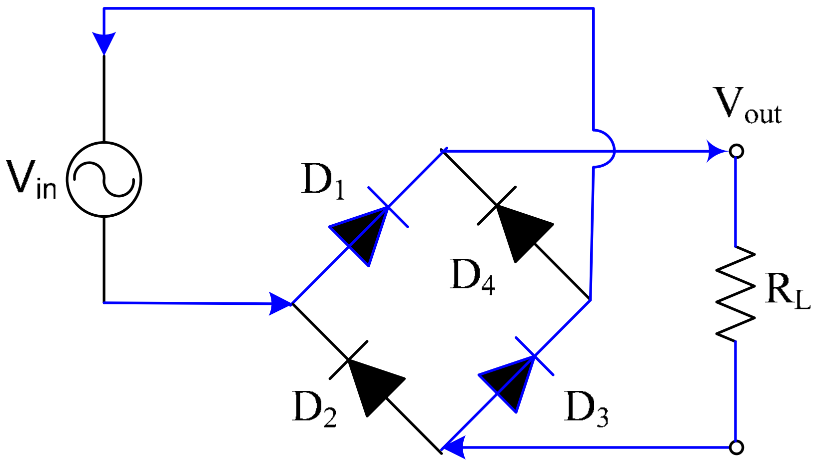


For the V_{in} positive half cycle, the current path is:



D_2 and D_4 are on while D_1 and D_3 are off. So V_{out} on the load resistor is V_{in} positive half cycle.

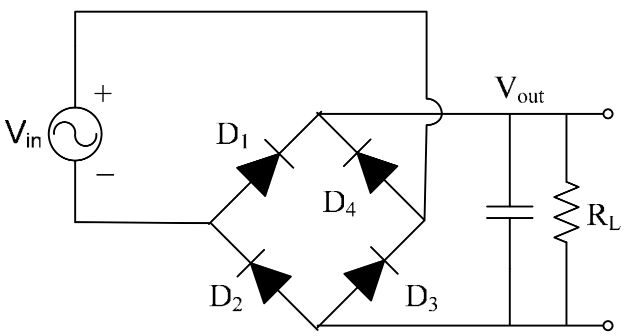
For the Vin negative half cycle, the current path is:

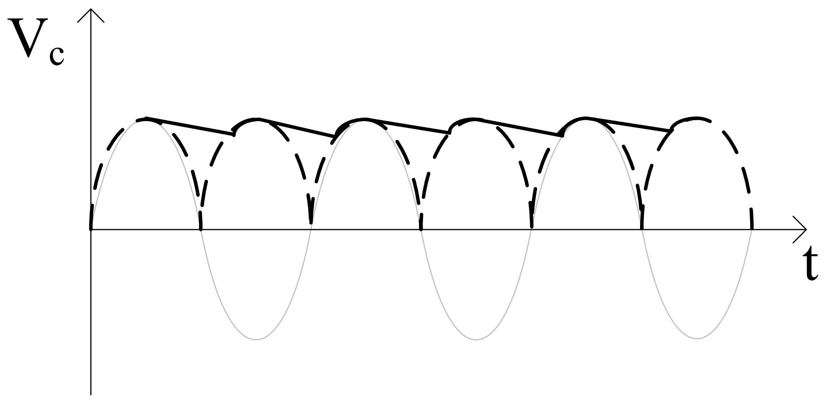


D_1 and D_3 are on while D_2 and D_4 are off. However, the current goes through the resistor in the same direction. So V_{out} on the load resistor for the negative half cycle is the same as it was for the positive half cycle.

So we get V_{out} looks like $|\sin \omega t|$. (Flip the negative half cycle.)

(b) If we put a large capacitor in stead of the resistor at the output as shown below, what is the voltage of the capacitor? Please draw the output you got in (a) as well for a reference and give explanations. [8 pts]





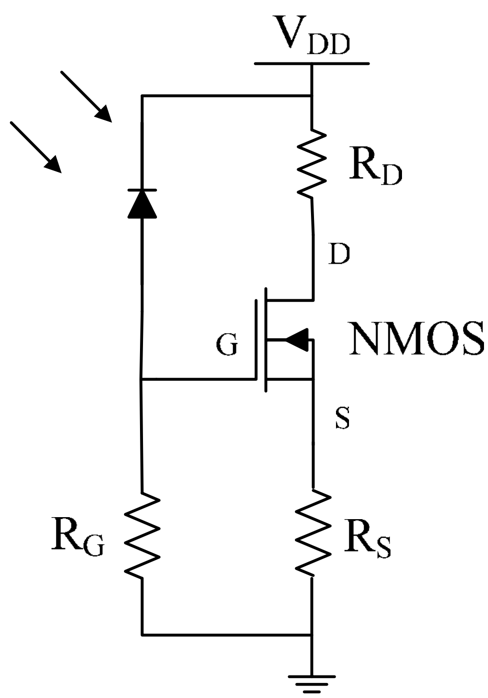
Consider the steady state.

The capacitor is charged up to the peak value when the input signal reaches its maximum. Then the input signal starts to drop, thus the capacitor is discharging. Since it's a large capacitor, it is being discharged slowly. By the time the absolute value of the input voltage is higher than it is on the capacitor, it's being charged again, which makes V_c to the peak value again.

(c) Which one do you think is more efficient in getting DC power, (a) or (b)? Why? [4 pts]

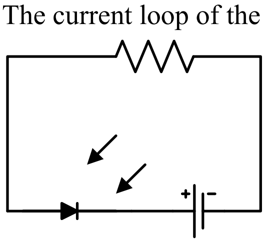
(b) is more efficient, since the average power is higher. Or area below the output is larger.

4. [30 pts] A photodetector (reverse biased photodiode) is connect to an NMOSFET as shown in the figure. We know V_{DD}=5 V, R_G=10 kΩ, R_D=50 Ω and R_S=200 Ω.



(a) The photodetector I-V characteristics are known as shown in the I_p-V_p plot below. When it is dark, it follows the top curve, while when it is under a certain amount of light, it shifts down. Now given that the light intensity is exactly the amount that makes the photodiode follow the bottom curve, please draw the load line of the photodiode on the I_p-V_p plot below, label its operating point and give the V_{pQ}, I_{pQ} values of the operating point. [10 pts]

The current loop of the photodetector is

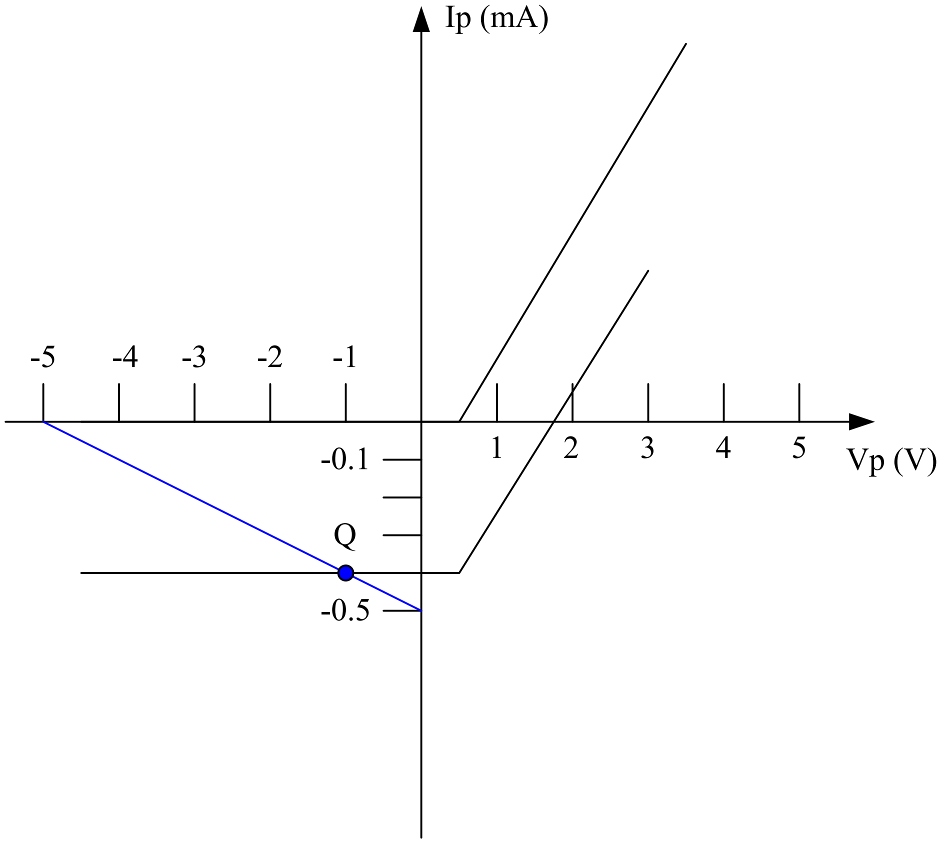


So the load line equation is: $V_{DD} = -V_p - I_p R_G$. Note that we put negative sign before V_p and

I_p, because the photodiode is reverse biased. Put the values of V_{DD} and R_G in, we get $5 = -V_p - I_p 10k$ and we draw this on the plot.

Since we know from the characteristics that the photocurrent now is -0.4 mA under reverse bias, so I_p= -0.4 mA and from the load line equation, we know immediately that V_p= -1V.

V_{pQ}= -1V	I_{pQ}= -0.4mA



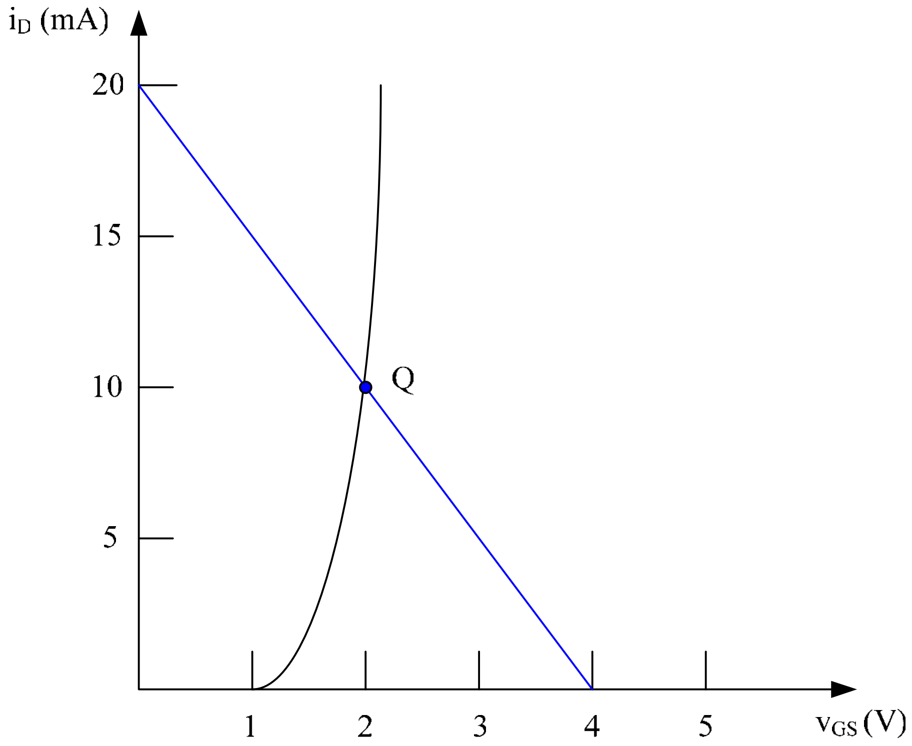
(b) Given the v_{GS}-i_D characteristics of the MOSFET which is shown in the v_{GS}-i_D plot below, please draw the load line on this plot, label the operating point Q of the MOSFET and give v_{GSQ} and i_{DQ} values of the operating point. [10 pts]

From the circuit configuration, we apply the KVL at gate and source and get the load line equation

$$v_{GS} = R_G I_p - i_D R_S = 10k \times 0.4m - i_D 200 = 4 - 200 i_D \Rightarrow v_{GS} = 4 - 200 i_D$$

We draw this on the v_{GS}-i_D plot. And from the MOSFET characteristics, we read the values of the operating point.

V_{GSQ}= 2 V	i_{DQ}= 10 mA



(c) Given the v_{DS}-i_D characteristics of the MOSFET which is shown in the v_{DS}-i_D plot below, please draw the load line on this plot, label the operating point Q of the MOSFET and give v_{DSQ} value of the operating point. [10 pts]

Apply KVL from VDD through drain and source to the ground.

We get the load line equation $V_{DD} = i_D R_D + v_{DS} + i_D R_s = i_D(200 + 50) + v_{DS}$

$$v_{DS} = V_{DD} - 250 i_D = 5 - 250 i_D$$

We draw this line on the plot below. Since the VGS is 2 V, the Q point will be on the middle curve. Since we know from question (b) that i_D is 10 mA, so v_{DS}=2.5 V

V_{DSQ}= 2.5 V