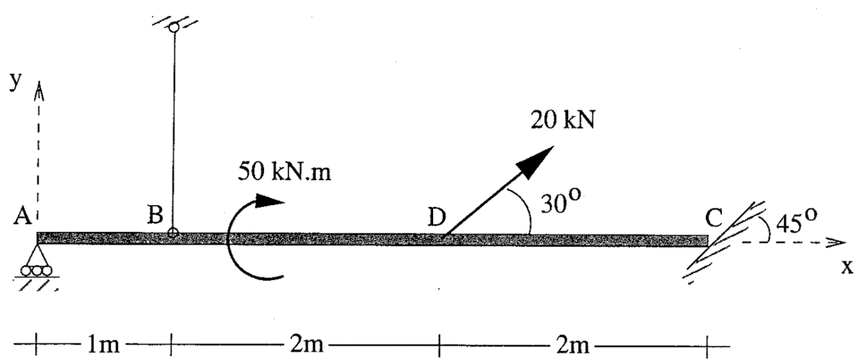
Problem 1 (10+10+15 points)

The bar shown in the figure below is supported by a roller at point A and a truss element at point B, and rests on a frictional surface at point C. The bar is subjected to the external force and moment shown in the figure, as well as to its own weight $W = 10$kN.



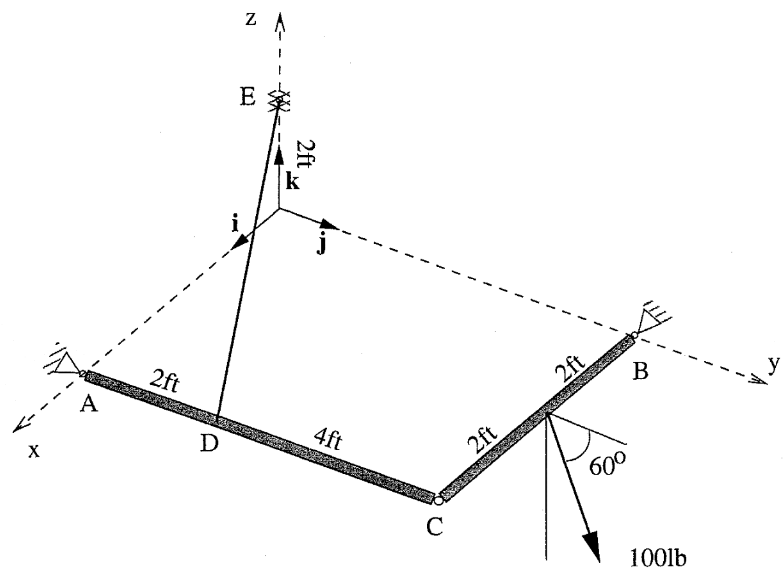
(a) Draw the free-body diagram of the bar showing all external forces.

(b) If friction is neglected, determine the magnitude of the force exerted on the bar by the truss element.

(c) If friction is included and sliding at point C is imminent, determine again the force exerted on the bar by the truss element. Assume that the static friction coefficient is $\mu_s = 0.5$.

Problem 2 (10+20 points)

The two rods AC and BC are hinged together at C and are supported by the cable DE and the ball-and-socket joints at A and B. Rod BC is subjected to a force on the plane normal to the x-axis.



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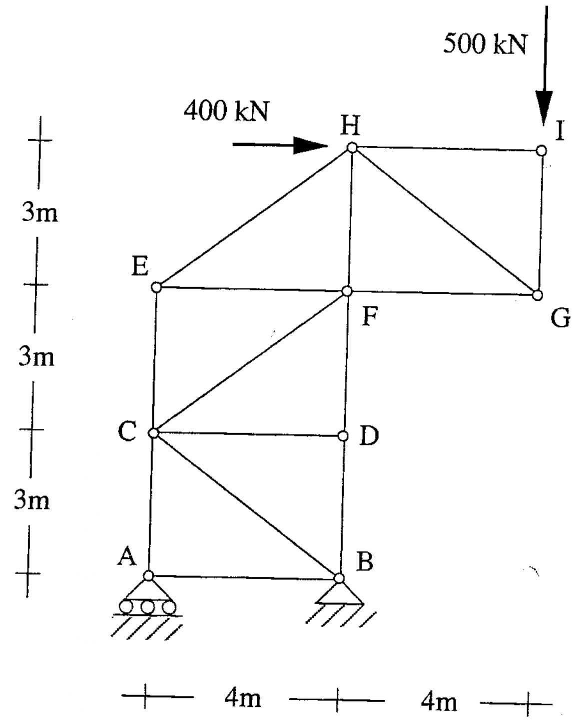
(a) Draw the free-body diagram of the system of the two rods.

(b) Using the free-body diagram of part (a), determine the tension T in the cable.
Hint: This can be accomplished by writing a single equilibrium equation.

Problem 3 (10+10+15 points)

Consider the planar truss structure shown in the figure below.



(a) Find the force in the truss member IG.

(b) Find the force in the truss member HI.

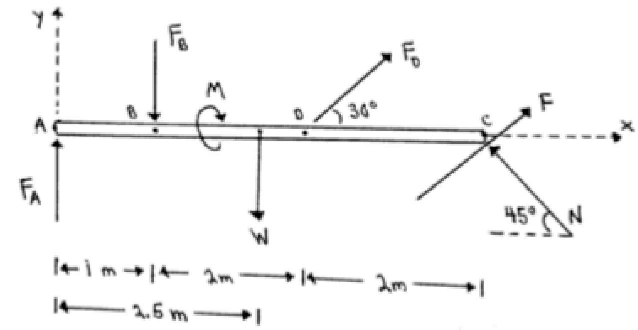
(c) Find the force in the truss member AC.

Mean score: 71

Midterm exam 1 solutions

Problem 1

a.



$M = 50$ kN·m

$F_D = 20$ kN

$W = 10$ kN

$\mu_s = 0.5$

b. neglect friction $\Rightarrow$ $F = 0$

ΣF_x: $\quad F_D\left(\frac{\sqrt{3}}{2}\right) - N\left(\frac{\sqrt{2}}{2}\right) = 0 \quad \longrightarrow \quad N = \sqrt{\frac{3}{2}}\,F_D = 10\sqrt{6}$ kN

$\circlearrowleft + \Sigma M_A$: $\quad -F_B(1) - M - W\left(\frac{5}{2}\right) + \left(\frac{1}{2}F_D\right)(3) + \left(\frac{\sqrt{2}}{2}N\right)(5) = 0$

$$F_B = -50 - 25 + 30 + 50\sqrt{3}$$

$$\boxed{F_B = 50\sqrt{3} - 45 \text{ kN}}$$

c. slip condition: $\quad F = \mu_s N$

ΣF_x: $\quad F_D\left(\frac{\sqrt{3}}{2}\right) - N\left(\frac{\sqrt{2}}{2}\right) + F\left(\frac{\sqrt{2}}{2}\right) = 0$

$\sqrt{3}\,F_D = \sqrt{2}\,(1-\mu_s)N = \frac{\sqrt{2}}{2}N \quad \longrightarrow \quad N = \sqrt{6}\,F_D = 20\sqrt{6}$ kN

$\circlearrowleft + \Sigma M_A$: $\quad -F_B(1) - M - W\left(\frac{5}{2}\right) + \left(\frac{1}{2}F_D\right)(3) + \left(\frac{\sqrt{2}}{2}N\right)(5) + \left(\frac{\sqrt{2}}{2}\mu_s N\right)(5) = 0$

$$F_B = -45 + 50\sqrt{12} + 25\sqrt{12}$$

$$\boxed{F_B = 150\sqrt{3} - 45 \text{ kN}}$$

Problem 1

c. alternative

if it was assumed the bar was slipping up the incline, the direction of F would be reversed in the free-body diagram (a)

under this assumption,

slip condition: $F = \mu_s N$

ΣF_x: $F_D\left(\frac{\sqrt{3}}{2}\right) - N\left(\frac{\sqrt{2}}{2}\right) - F\left(\frac{\sqrt{2}}{2}\right) = 0$

$\sqrt{3}\,F_D = \sqrt{2}(1+\mu_s)N = \frac{3\sqrt{2}}{2}N \quad \longrightarrow \quad N = \frac{\sqrt{6}}{3}F_D = \frac{20}{3}\sqrt{6}$ KN

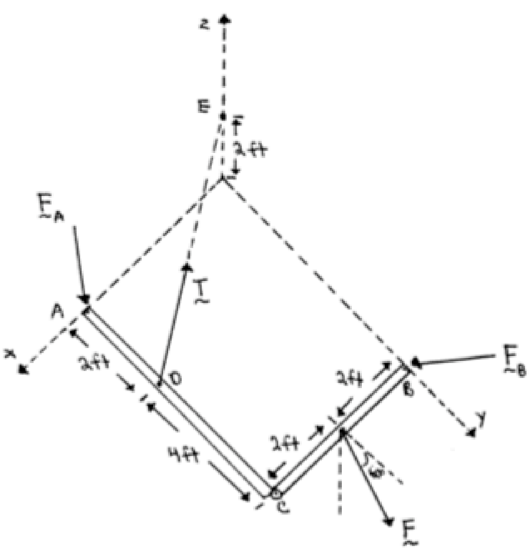
$\circlearrowleft + \Sigma M_A$: $-F_B(1) - M - W\left(\frac{5}{2}\right) + \left(\frac{1}{2}F_D\right)(3) + \left(\frac{\sqrt{2}}{2}N\right)(5) - \left(\frac{\sqrt{2}}{2}\mu_s N\right)(5) = 0$

$F_B = -45 + \frac{50}{3}\sqrt{12} - \frac{25}{3}\sqrt{12}$

$$\boxed{F_B = \frac{50}{3}\sqrt{3} - 45 \text{ KN}}$$

Problem 2

a.



$\underset{\sim}{F} = 50\,\underset{\sim}{j} - 50\sqrt{3}\,\underset{\sim}{K}$

$\underset{\sim}{F}_A = F_{Ax}\,\underset{\sim}{i} + F_{Ay}\,\underset{\sim}{j} + F_{Az}\,\underset{\sim}{K}$

$\underset{\sim}{F}_B = F_{Bx}\,\underset{\sim}{i} + F_{By}\,\underset{\sim}{j} + F_{Bz}\,\underset{\sim}{K}$

$\underset{\sim}{T} = T\,\underset{\sim}{u}_{DE}$

b.

$$\underset{\sim}{T} = T\,\underset{\sim}{u}_{DE} = T\frac{\underset{\sim}{r}_{DE}}{r_{DE}} \qquad \text{where} \qquad \underset{\sim}{r}_{DE} = \underset{\sim}{r}_E - \underset{\sim}{r}_D = 2\underset{\sim}{k} - (4\underset{\sim}{i} + 2\underset{\sim}{j})$$

$$r_{DE} = \sqrt{2^2 + (-4)^2 + (-2)^2} = 2\sqrt{6}$$

$$\longrightarrow \quad \underset{\sim}{T} = \frac{T}{\sqrt{6}}(-2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k})$$

moments about point A: $\underset{\sim}{M} = \underset{\sim}{r}_{AD} \times \underset{\sim}{T} + \underset{\sim}{r}_{AF} \times \underset{\sim}{F} + \underset{\sim}{r}_{AB} \times \underset{\sim}{B} = \underset{\sim}{0}$

moment along axis AB: $M_{AB} = \underset{\sim}{u}_{AB} \cdot \underset{\sim}{M} = \underset{\sim}{u}_{AB} \cdot (\underset{\sim}{r}_{AD} \times \underset{\sim}{T}) + \underset{\sim}{u}_{AB} \cdot (\underset{\sim}{r}_{AF} \times \underset{\sim}{F}) + \underset{\sim}{u}_{AB} \cdot (\underset{\sim}{r}_{AB} \times \underset{\sim}{F})$

where,

$$\underset{\sim}{u}_{AB} = \frac{\underset{\sim}{r}_B - \underset{\sim}{r}_A}{|\underset{\sim}{r}_B - \underset{\sim}{r}_A|} = \frac{6\underset{\sim}{j} - 4\underset{\sim}{i}}{\sqrt{52}} = -\frac{2}{\sqrt{13}}\underset{\sim}{i} + \frac{3}{\sqrt{13}}\underset{\sim}{j}$$

$$\underset{\sim}{r}_{AD} = 2\underset{\sim}{j}$$

$$\underset{\sim}{r}_{AF} = -2\underset{\sim}{i} + 6\underset{\sim}{j}$$

note: $\underset{\sim}{u}_{AB}$ is parallel to $\underset{\sim}{r}_{AB}$ so $\underset{\sim}{u}_{AB} \cdot (\underset{\sim}{r}_{AB} \times \underset{\sim}{F}) = \underset{\sim}{0}$

[$\underset{\sim}{F}_B$ passes through axis AB at B so creates no moment about the axis]

$$M_{AB} = \begin{vmatrix} -\frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} & 0 \\ 0 & 2 & 0 \\ -\frac{2}{\sqrt{6}}T & -\frac{1}{\sqrt{6}}T & \frac{1}{\sqrt{6}}T \end{vmatrix} + \begin{vmatrix} -\frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} & 0 \\ -2 & 6 & 0 \\ 0 & 50 & -50\sqrt{3} \end{vmatrix} = 0$$

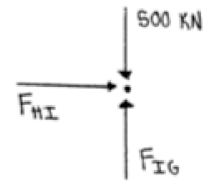
$$-\frac{2}{\sqrt{13}}\left[2\left(\frac{1}{\sqrt{6}}T\right) - 0\right] - \frac{2}{\sqrt{13}}\left[6(-50\sqrt{3}) - 0\right] - \frac{3}{\sqrt{13}}\left[(-2)(-50\sqrt{3}) - 0\right] = 0$$

$$-\frac{4}{\sqrt{6}}T + 600\sqrt{3} - 300\sqrt{3} = 0$$

$$\boxed{T = 225\sqrt{2}\ \text{lb}}$$

Problem 3

a. joint I:



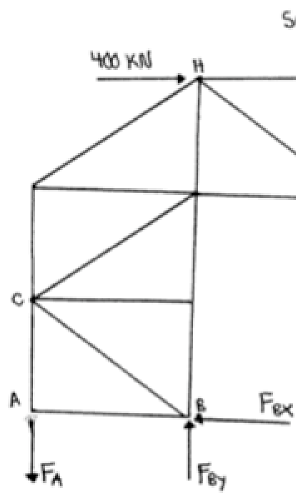
ΣF_y: $F_{IG} - 500 = 0$

$\boxed{F_{IG} = 500 \text{ KN} \ (C)}$

b.

ΣF_x: $\boxed{F_{HI} = 0}$

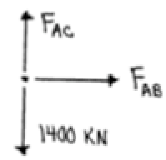
c.



$\circlearrowleft + \Sigma M_B$: $F_A(4) - 400(9) - 500(4) = 0$

$F_A = 1400 \text{ KN}$

joint A:



ΣF_y: $F_{AC} - 1400 = 0$

$\boxed{F_{AC} = 1400 \text{ KN} \ (T)}$