ECS 40, Fall 2008
Prof. Chang-Hasnain
Test #2
Version A

10:10 am – 11:00 am, Monday October 27, 2008
Total Time Allotted: 50 minutes
Total Points: 100

1. This is a closed book exam. However, you are allowed to bring one page (8.5" x 11"), double-sided notes.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
4. **Remember to put down units. Points will be taken off for missed unit.**

Last (Family) Name:____________________________________

First Name: ____

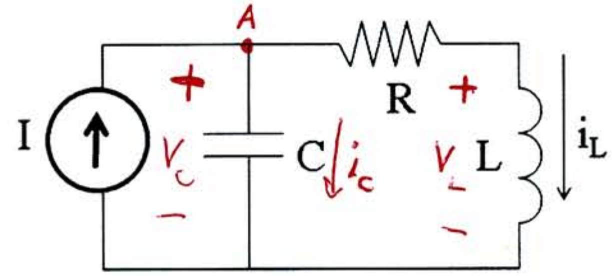
Student ID: __________________ Discussion Session: __________

Signature: ____

Score:	
Problem 1 (40 pts)	
Problem 2 (24 pts)	
Problem 3 (16 pts)	
Problem 4 (20 pts)	
Total	

Problem 1: Transient Response of RLC circuits (40 pts)

Consider the circuit shown below



a) Derive the differential equation for $i_L(t)$ (20 pts)

$$KVL: V_c = Ri_L + V_L = Ri_L + L\frac{di_L}{dt} \quad (1)$$

$$KCL@A: I - i_L = i_c = C\frac{dV_c}{dt} \quad (2)$$

Differentiate (1)

$$\frac{dV_c}{dt} = R\frac{di_L}{dt} + L\frac{d^2 i_L}{dt^2} \quad (3)$$

Plug (3) → (2)

$$I - i_L = CR\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2}$$

Rearranging

$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = \frac{I}{LC}$$

Solution: $\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = \frac{I}{LC}$

b) Let L = 1 H, C = 1/3 F, R = 4 Ω, I = 2A. What is the undamped resonant frequency ω_0? It is ok to express your answer as a square root. (5 pts)

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1)(1/3)}}$$

$$\omega_0 = \sqrt{3}\ \frac{rad}{sec}$$

$\boxed{\omega_0 = \sqrt{3}\ \frac{rad}{sec}}$

c) What is the damping ratio ζ? Is this system over-, critically, or under- damped? Again, it is ok to have square roots in your answer. (10 pts)

$$2\zeta\omega_0 = R/L$$

$$\zeta = \frac{R}{2L\omega_0} = \frac{4}{2(1)(\sqrt{3})} = \frac{2}{\sqrt{3}}$$

$\boxed{\zeta = \frac{2}{\sqrt{3}}}$

$\zeta = \frac{2}{\sqrt{3}} > 1 \Rightarrow$ overdamped

- ☒ overdamped
- O critically damped
- O underdamped

d) Write down the general form of the solution. Give numerical values for the forced response . You don't need to solve for any other constant (K_1, K_2, $s_{1,2}$ etc.) (5 pts)

Solution for overdamped case:

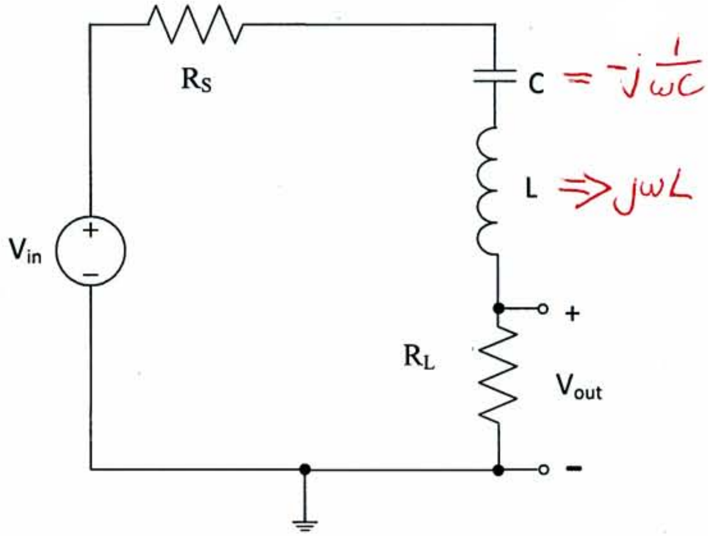
$$i_L(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + f(t)$$

@ steady-state $\frac{d^2 i_L}{dt^2} = \frac{d^2 i_L}{dt} = 0 \Rightarrow \frac{1}{LC} i_L = \frac{1}{LC} I \Rightarrow i_L(t) = I = 2\ [A]$

$\boxed{i_L(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + 2\ [A]}$

Problem 2: 2^{nd}-Order Circuit (24 points)

Consider the circuit shown below



(a) Using complex impedances, derive the transfer function $H(\omega)=V_{out}/V_{in}$ as a function of R_s, C, L and R_L. (12 pts)

Bring your answer into one of the standard forms known from lecture (high-pass, low-pass band-pass, band-reject (notch) filter)

$$H(\omega) = \frac{R_L}{R_L+R_S+j\left(\omega L - \frac{1}{\omega C}\right)}$$

Series RLC $\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

$$H(\omega) = \frac{R_L}{R_L+R_S+j\left(\frac{\omega L\frac{1}{\sqrt{LC}}}{\frac{1}{\sqrt{LC}}} - \frac{\frac{1}{\sqrt{LC}}}{\omega C\frac{1}{\sqrt{LC}}}\right)} = \frac{R_L}{R_L+R_S+j\left(\frac{\omega\sqrt{L/C}}{\frac{1}{\sqrt{LC}}} - \frac{\frac{1}{\sqrt{LC}}}{\omega\sqrt{\frac{C}{L}}}\right)}$$

$$H(\omega) = \frac{R_L}{R_L+R_S+j\sqrt{\frac{L}{C}}\left(\frac{\omega}{\frac{1}{\sqrt{CL}}} - \frac{\frac{1}{\sqrt{LC}}}{\omega}\right)} = \frac{\frac{R_L}{R_L+R_S}}{1+j\frac{1}{R_L+R_S}\sqrt{\frac{L}{C}}\left(\frac{\omega}{\sqrt{\frac{1}{LC}}} - \frac{\sqrt{\frac{1}{LC}}}{\omega}\right)}$$

$$H(\omega) = \frac{K}{1+jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$\boxed{H(\omega) = \frac{\frac{R_L}{R_L+R_S}}{1+j\frac{1}{R_L+R_S}\sqrt{\frac{L}{C}}\left(\frac{\omega}{\frac{1}{\sqrt{LC}}} - \frac{\frac{1}{\sqrt{LC}}}{\omega}\right)}}$$

(b) What type of filter does this circuit implement? (3 pts)

O Low-pass	O High-pass
O Band-pass	O Band-reject (Notch)

(c) Depending on your answer in part b), answer one of the following 3 questions ((i) – (iii)) (9 pts)

(i) If your answer in part b) is either high- or low-pass filter:
Write down the break frequency ω_B, the slope of $|H(\omega)^2|$ in dB/decade for very small and very high frequencies respectively as well as the order of the filter.

$\omega_B =$

slope =

order =

(ii) If your answer in part b) is band-pass filter:

Write down the resonance frequency ω_0, the quality factor Q of the filter as well as the slope of $|H(\omega)^2|$ in dB/decade for very small and very high frequencies.

$\omega_0 = \frac{1}{\sqrt{LC}}$

$Q = \frac{1}{R_L + R_S}\sqrt{\frac{L}{C}}$

slope (ω<<) = $20\ \frac{dB}{dec}$

slope (ω>>) = $-20\ \frac{dB}{dec}$

(iii) If your answer in part b) is band-reject (notch) filter:

Write down the resonance frequency ω_0, the quality factor Q of the filter as well as the values of $|H(\omega)^2|$ in dB for very small and very high frequencies.

$\omega_0 =$

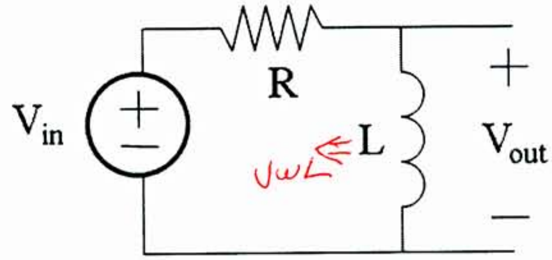
$Q =$

$|H(\omega<<)| =$

$|H(\omega>>)| =$

Problem 3: 1st Order Bode Plots (16 pts)

Consider the circuit shown below



a) Using complex impedances, derive the transfer function $H(\omega)=V_{out}/V_{in}$ as a function of R and L (6 pts)

$$H(\omega) = \frac{j\omega L}{R+j\omega L} = \frac{1}{\frac{R}{j\omega L}+1} = \frac{1}{1-j\frac{R}{\omega L}}$$

$$\boxed{H(\omega) = \frac{1}{1-j\frac{R}{\omega L}}}$$

b) What is ω_B or ω_0 (depending on the transfer function you came up with in a)) of the system? (Give your answer again as a function of R and L) (2 pts)

$$\frac{\omega_0}{\omega} = \frac{R}{\omega L}$$

$$\omega_0 = \frac{R}{L}$$

$$\boxed{\omega_B \text{ or } \omega_0 = R/L}$$

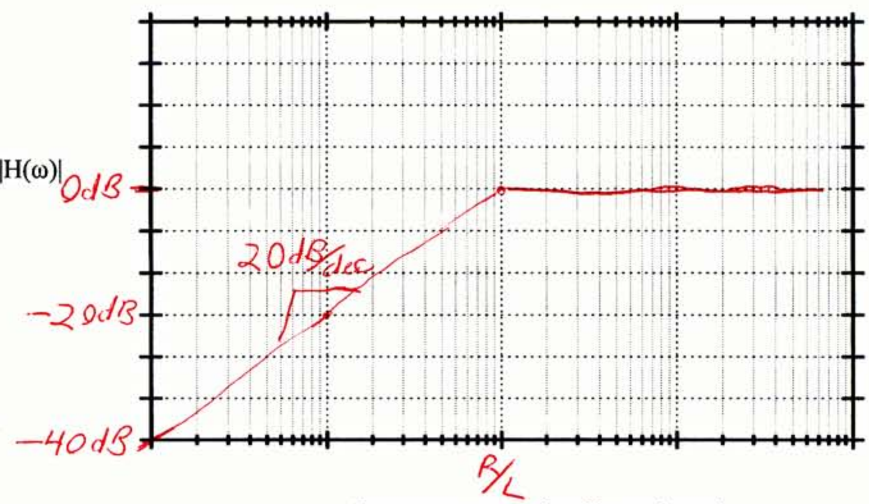
c) What type of filter does this circuit implement? (2 pts)

O Low-pass	~~O~~ High-pass
O Band-pass	O Band-reject (Notch)

d) Sketch the magnitude Bode plot on the graph below. Make sure you mark all important characteristics of your graph (important values of ω, slopes, pass-band value). (6 pts)

Be aware of the fact that the frequency axis is logarithmic

20log|H(ω)|



frequency w (radians/sec)

Problem 4: Bode Plots (20 pts)

Given is the following transfer function:

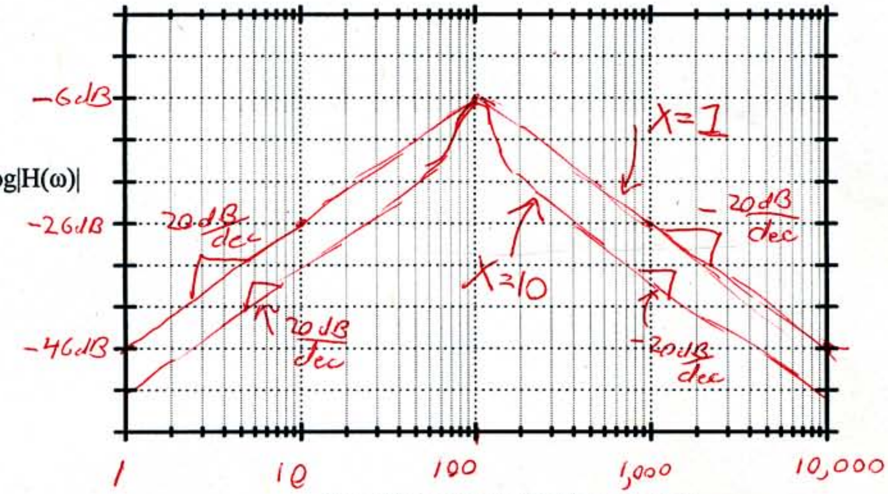
$$H(\omega) = \frac{K}{1 + jX\left(\frac{\omega}{Y} - \frac{Y}{\omega}\right)}$$

where K = 0.5 and Y = 100

In the graphs below sketch the amplitude and phase transfer functions for the two cases where X = 1 and X = 10. Make sure you clearly mark important characteristics (any slopes of magnitude response as well as magnitude and phase at ω=Y) in your graphs and you clearly indicate any differences between the two cases.

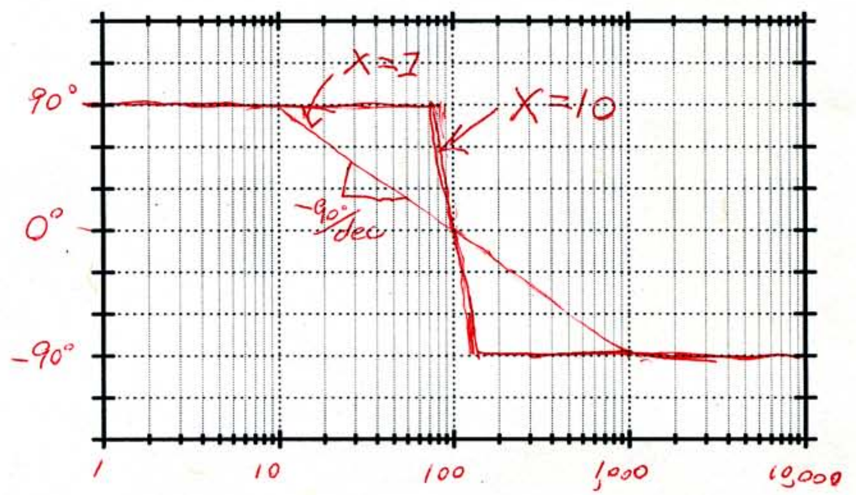
Be aware of the fact that the frequency axis is logarithmic.

20log|H(ω)|



frequency w (radians/sec)

Phase of H(w) (degrees)



frequency w (radians/sec)