EECS 40, Spring 2006
Prof. Chang-Hasnain
Midterm #2

April 6, 2006
Total Time Allotted: 80 minutes
Total Points: 100

1. This is a closed book exam. However, you are allowed to bring two pages (8.5” x 11”), double-sided notes
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
4. Draw BOXES around your final answers.
5. **Remember to put down units.** Points will be taken off for answers without units.
6. **NOTE: $\mu=10^{-6}$; $k=10^{3}$; $M=10^{6}$**

Last (Family) Name:____

First Name: ____

Student ID: ____________________Lab Session: _________Dis. Session: _____

Signature: ____

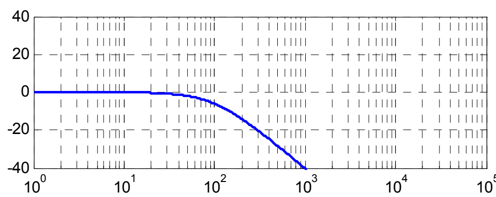
Score:	
Problem 1 (20 pts)	
Problem 2 (35 pts):	
Problem 3 (15 pts):	
Problem 4 (30 pts):	
Total	

1.(20 pts) Match the transfer function to the Bode plot. Each transfer function matches to exactly one Bode plot. Also, there is no partial credit for this question.

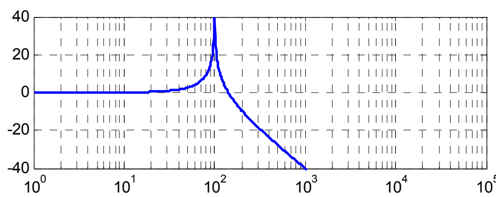
a. $H(f)=\dfrac{1}{\left(\frac{jf}{100}\right)^2+0.5\left(\frac{jf}{100}\right)+1}$	b. $H(f)=\dfrac{1}{\left(\frac{jf}{100}+1\right)^2}$	c. $H(f)=\dfrac{1}{\frac{jf}{1000}+1}$
d. $H(f)=\dfrac{1}{\frac{jf}{100}+1}$	e. $H(f)=\dfrac{1}{\left(\frac{jf}{100}\right)^2+1}$	

Mark your answer here Magnitude Plot (dB)

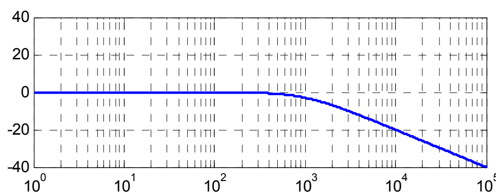
b



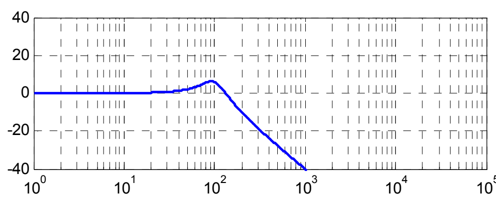
e



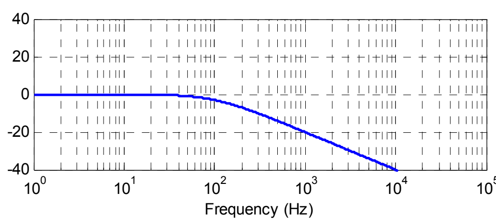
c



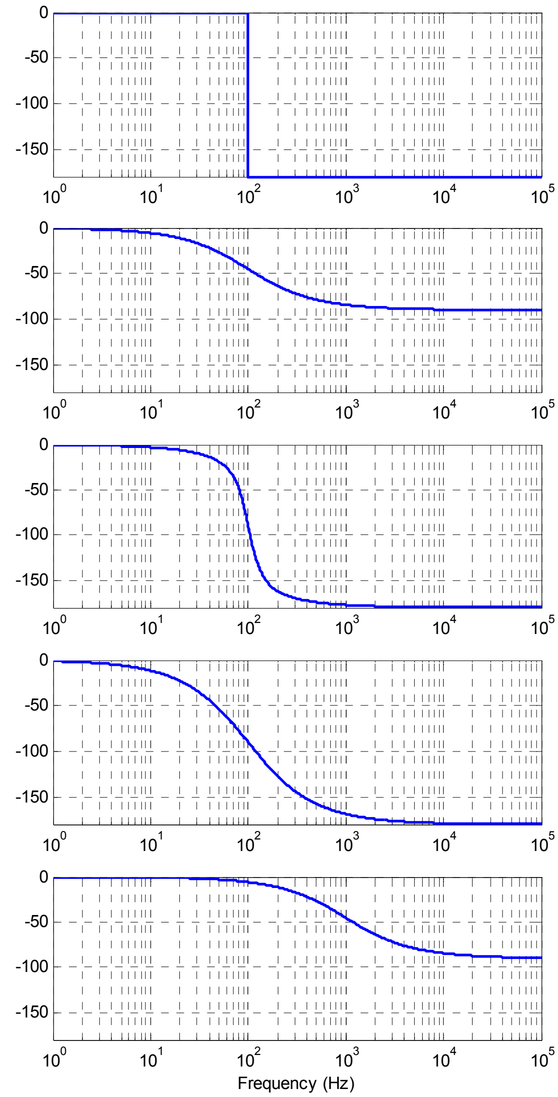
a



d



Mark your answer here	Phase Plot (degrees)
e	
d	
a	
b	
c	



For the magnitude plot, we first split the list into first- and second-order Bode plots. The first order Bode plots have a -20dB/decade slope, and the second-order Bode plots have a -40dB/decade slope.

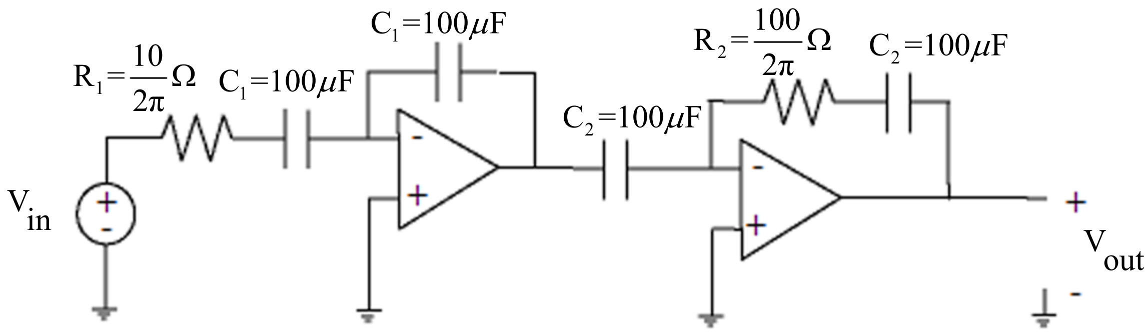
Looking at the breakpoints of the first-order Bode plots, we see that (c) has a breakpoint at f = 1000Hz, and that (d) has a breakpoint at f = 100Hz.

Looking at the size of the humps/peaks of the second-order Bode plots, decreasing values of zeta gives rise to a larger peak. (Note that technically speaking, (b) is two first-order terms but we can think of it as having a zeta = 1).

From this, we have that the magnitude plots match as: (b), (e), (c), (a), (d).

For the phase plot, we again split the list into first- and second-order terms. For first-order terms, the phase plot is -45 degrees at the breakpoint. For second order terms, decreasing values of zeta gives rise to a sharper phase transition.

2.(35 pts) The circuit schematic for a functional block known as a lead compensator is:



a (15 pts) Let R_1 = 10/(2π) ohms, R_2 = 100/(2π) ohms, C_1 = 100 uF, and C_2 = 100 uF. Show that the transfer function of the circuit shown above is:

$$H(\mathrm{f}) = \frac{\frac{\mathrm{jf}}{100}+1}{\frac{\mathrm{jf}}{1000}+1}$$

Method 1: all at once

1) $V_{in} - I_1\left(R_1 + \frac{1}{j\omega C_1}\right) = 0$

2) $0V - I_1\left(\frac{1}{j\omega C_1}\right) - I_2\left(\frac{1}{j\omega C_2}\right) = 0$; $I_1\left(\frac{1}{j\omega C_1}\right) = I_2\left(\frac{1}{j\omega C_2}\right)$; $I_1 = I_2\left(\frac{j\omega C_1}{j\omega C_2}\right) = I_2\left(\frac{C_1}{C_2}\right)$

3) $0V - I_2\left(R_2 + \frac{1}{j\omega C_2}\right) = V_{out}$; $I_2 = \frac{-V_{out}}{\left(R_2 + \frac{1}{j\omega C_2}\right)}$

3+2) $I_1 = \left(\frac{C_1}{C_2}\right)\frac{-V_{out}}{\left(R_2 + \frac{1}{j\omega C_2}\right)}$

Combine with 1): $V_{in} - \left[\left(\frac{C_1}{C_2}\right)\frac{-V_{out}}{\left(R_2 + \frac{1}{j\omega C_2}\right)}\right]\left[\left(R_1 + \frac{1}{j\omega C_1}\right)\right] = 0$

$$-\left(\frac{C_2}{C_1}\right)\frac{\left(R_2+\frac{1}{j\omega C_2}\right)}{\left(R_1+\frac{1}{j\omega C_1}\right)}=\frac{V_{out}}{V_{in}};$$

$$\frac{V_{out}}{V_{in}}=-\frac{\left(R_2C_2+\frac{1}{j\omega}\right)}{\left(R_1C_1+\frac{1}{j\omega}\right)}=-\frac{\left(j\omega R_2C_2+1\right)}{\left(j\omega R_1C_1+1\right)}=-\frac{\left(j2\pi f\left(\frac{100}{2\pi}\Omega\right)100\mu F+1\right)}{\left(j2\pi f\left(\frac{10}{2\pi}\Omega\right)100\mu F+1\right)}=\frac{\left(j\left(\frac{f}{100}\right)+1\right)}{\left(j\left(\frac{f}{1000}\right)+1\right)}$$

Method 2: Two Inverters

$$\frac{Vout1}{Vin1}=-\frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1}+R_1}=-\frac{1}{1+j\omega R_1C_1}=-\frac{1}{1+j2\pi f(\frac{10}{2\pi})(100\mu F)}=-\frac{1}{1+j\frac{f}{1000}}$$

$$\frac{Vout2}{Vin2}=-\frac{\frac{1}{j\omega C_2}+R_2}{\frac{1}{j\omega C_2}}=-\frac{1+j\omega R_2C_2}{1}=-\left[1+j2\pi f(\frac{100}{2\pi})(100\mu F)\right]=-\left(1+j\frac{f}{100}\right)$$

$$\frac{Vout}{Vin}=-\frac{1}{1+j\frac{f}{1000}}\times-\left(1+j\frac{f}{100}\right)=\frac{j\frac{f}{100}+1}{j\frac{f}{1000}+1}$$

Method 3: Solve in stages:

$$\frac{Vin-0}{R_1+\frac{1}{j\omega C_1}}=\frac{0-Vout1}{\frac{1}{j\omega C_1}};\frac{Vout1}{Vin1}=-\frac{\frac{1}{j\omega C_1}}{R_1+\frac{1}{j\omega C_1}}$$ .. then same analysis as above

$$\frac{Vin2-0}{\frac{1}{j\omega C_2}}=\frac{0-Vout2}{R_2+\frac{1}{j\omega C_2}};\frac{Vout2}{Vin2}=-\frac{R_2+\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2}}$$ .. then same analysis as above

2b (12 pts) In the following table, write the magnitude and phase values for H(f) for f=100Hz, f=1000 Hz, very low f values ($f \to 0\,Hz$) and very high f values ($f \to \infty\,Hz$). These answers only need to be within 1.5 times the correct answer (but only because of rounding errors or sketching inaccuracies that you might have. Do not use the "straight line" approximation if it will cause your answer will be off from the exact value by more than 1.5 times).

Note – terms in red should be f, not ω. Was announced during midterm

f value (Hz)	**10 log |H(ω)|^2**	**$\angle$H(ω)**
Very low f ($f \to 0\,Hz$)	3dB	39.7 deg
f = 100Hz	17dB	39.7 deg
f = 1000Hz	0dB	0 deg
Very high f ($f \to \infty\,Hz$)	20dB	0 deg

Given terms:

$\tan^{-1}(0.1) = 5.7$ deg
$\tan^{-1}(0.5) = 26.6$ deg
$\tan^{-1}(1) = 45$ deg
$\tan^{-1}(2) = 63.4$ deg
$\tan^{-1}(10) = 84.3$ deg

Magnitude: $10\log|H(f)|^2 = 10\log\left[\frac{\left(1+\frac{f^2}{10^4}\right)}{\left(1+\frac{f^2}{10^6}\right)}\right] = 10\log\left[\left(1+\frac{f^2}{10^4}\right)\right] - 10\log\left[\left(1+\frac{f^2}{10^6}\right)\right]$

Phase: $\tan^{-1}\left(\frac{f}{100}\right) - \tan^{-1}\left(\frac{f}{1000}\right)$

For f=100Hz, becomes 3dB – 0 dB = 3dB
For f=1000Hz, becomes 20dB – 3dB = 17dB
Low f becomes 0dB – 0dB

High f becomes $10\log\left[\frac{\left(\frac{f^2}{10^4}\right)}{\left(\frac{f^2}{10^6}\right)}\right] = 10\log[100] = 20dB$

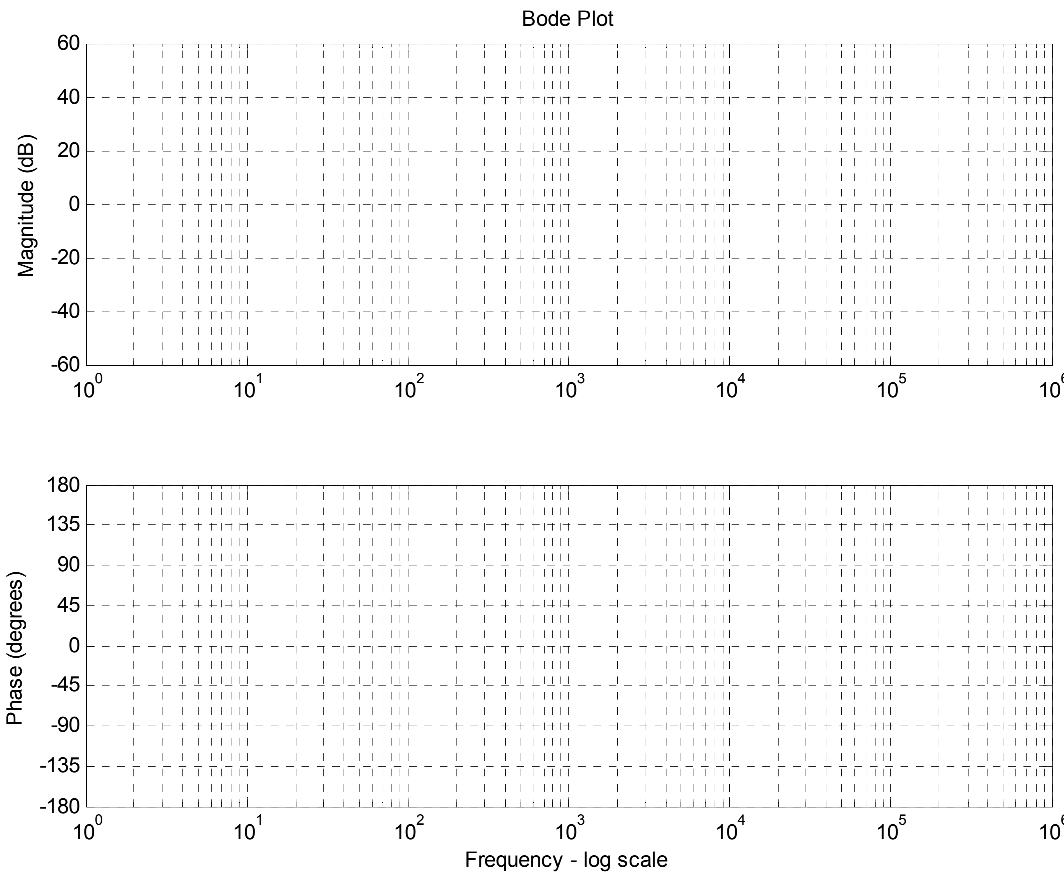
For f=100Hz, becomes $\tan^{-1}(1) - \tan^{-1}(.1) = 45° - 5.7° = 39.3°$

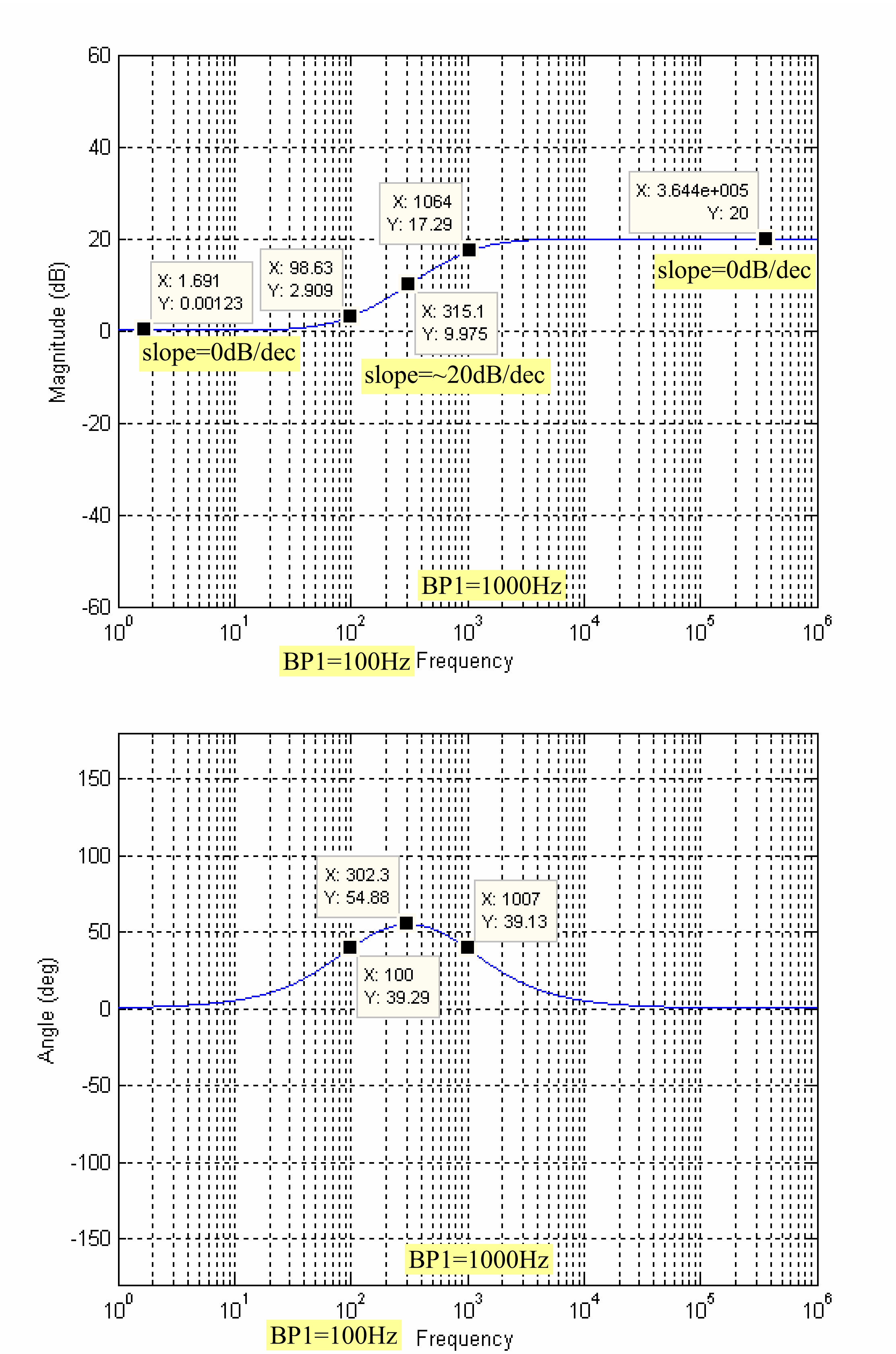
For f=1000Hz, becomes $\tan^{-1}(10) - \tan^{-1}(1) = 84.3° - 45° = 39.3°$

For f-> 0, becomes $\tan^{-1}(0) - \tan^{-1}(0) = 0°$

For f-> infinity, becomes $\tan^{-1}(\infty) - \tan^{-1}(\infty) = 90° - 90° = 0°$

2c (8 pts) Sketch the Bode plot of this transfer function. Sketch BOTH the magnitude and phase plot. Make sure to label the slopes of segments, the two break points of the transfer function, the low frequency magnitude, the high frequency magnitude, and the highest value on the phase plot. Be as accurate as you can, i.e., do not use the "straight line" approximation except as a starting guide if you wish for plotting the actual transfer function.

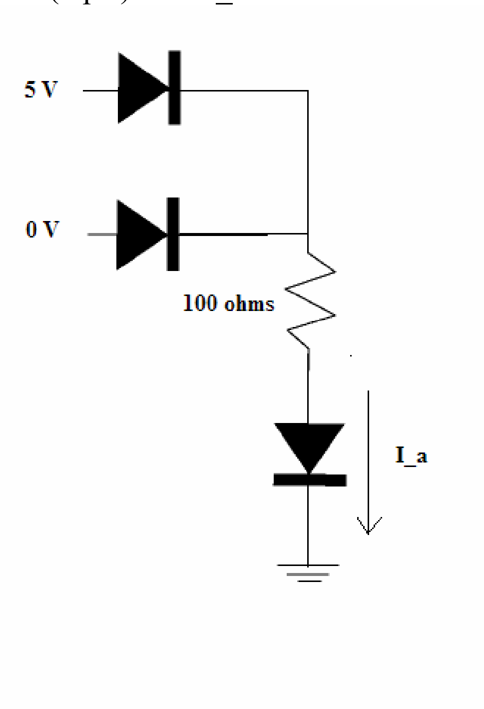



60
40
20
0
-20
-40
-60
Magnitude (dB)
X: 1.691
Y: 0.00123
X: 98.63
Y: 2.909
X: 1064
Y: 17.29
X: 315.1
Y: 9.975
X: 3.644e+005
Y: 20
slope=0dB/dec
slope=~20dB/dec
slope=0dB/dec
BP1=1000Hz
BP1=100Hz
Frequency
150
100
50
0
-50
-100
-150
Angle (deg)
X: 302.3
Y: 54.88
X: 100
Y: 39.29
X: 1007
Y: 39.13
BP1=1000Hz
BP1=100Hz
Frequency

3.(15 pts) Find the unknown values in the circuits below. For the diodes, use the "0.8V ON-OFF" model:

~~If I_d < 0, then the diode is open or OFF~~
If I_d = 0, then the diode is open or OFF
~~If I_d >= 0, then the diode is a 0.8V source or ON~~
If I_d > 0, then the diode is a 0.8V source or ON

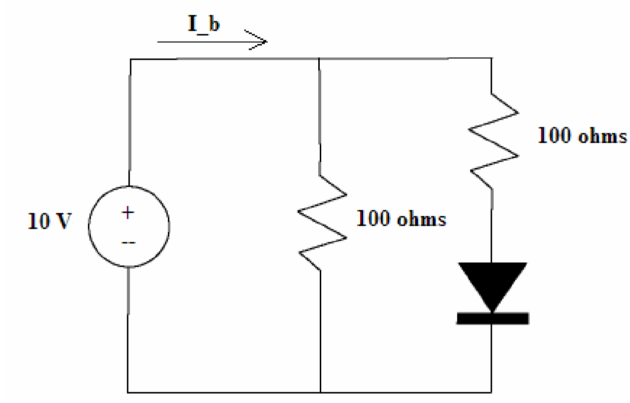
a. (5 pts) Find I_a in the circuit below:



0V diode is off, 5V diode and I_a diode are on.

$$I\_a = \frac{5V - 0.8V}{100\Omega} = 42mA$$

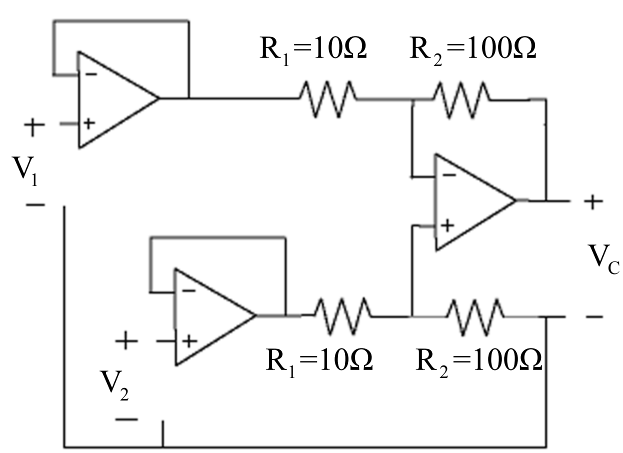
b. (5 pts) Find I_b in the circuit below:



1) 10V – I_1(100ohm) = 0;
   I_1=100mA (I_1 is current in left branch)
2) 10V – I_2(100ohm)-0.8V = 0
   I_2=92mA (I_2) is current in right branch with diode)

So I_b=I_1+I_2=192mA

c. (5 pts) Let R_1 = 10 ohms and R_2 = 100 ohms. Find V_c in the circuit below, in terms of V_1 and V_2:



1) V2 – I2(R1) – I2(R2) = 0
1) V2 – I2(R1+R2)=0

1) $\frac{V_2}{R_1 + R_2} = I_2$

2) V1 – I1(R1) – I1(R2) = Vc
2) V1 – I1(R1 + R2) = Vc
2) $\frac{V_1 - V_C}{R_1 + R_2} = I_1$

3) Vc + I1(R2) = I2(R2) (on far right opamp both voltages same)

3 + 2 + 1) Vc + $\frac{V_1 - V_C}{R_1 + R_2}$R2 = $\frac{V_2}{R_1 + R_2}$R2

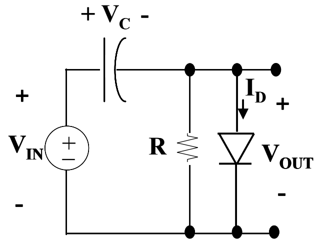
$V_c\left(1 - \frac{R_2}{R_1 + R_2}\right) = -\frac{R_2 V_1}{R_1 + R_2} + \frac{R_2 V_2}{R_1 + R_2}$; $V_c\left(\frac{R_1}{R_1 + R_2}\right) = \frac{R_2}{R_1 + R_2}\left(V_2 - V_1\right)$; $V_c = \frac{R_2}{R_1}\left(V_2 - V_1\right)$

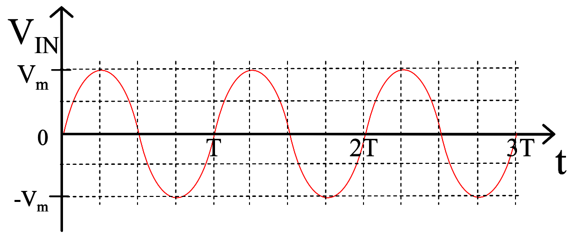
4.(30 pts) Consider the circuit shown below, in which the RC time constant is very long compared to the period *T* of the input $V_{IN}(t)$. Use the Ideal Diode model:

If $V_D < 0$, then the diode is OFF and does not pass current ($I_D=0$)

If $I_D >= 0$, then the diode is ON and $V_D=0$

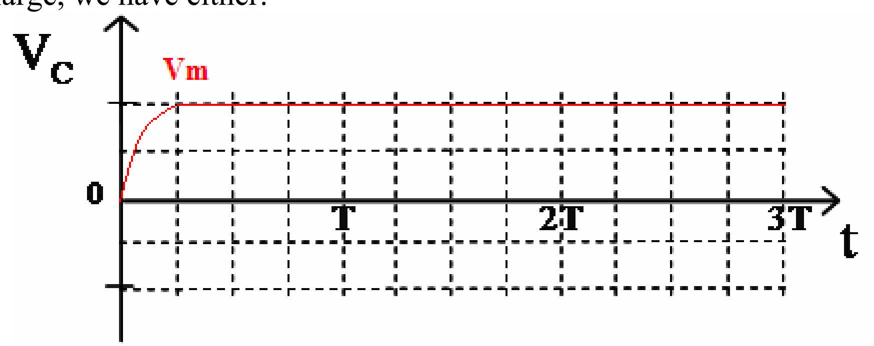
V_D is the voltage drop across the diode and I_D is current through the diode. $V_D = V_{OUT}$ in this problem. Analyze the following circuit. Given $V_{IN}(t)=V_m sin(2\pi t/T)$ for t>0, and $V_C(t=0^-)=0$.



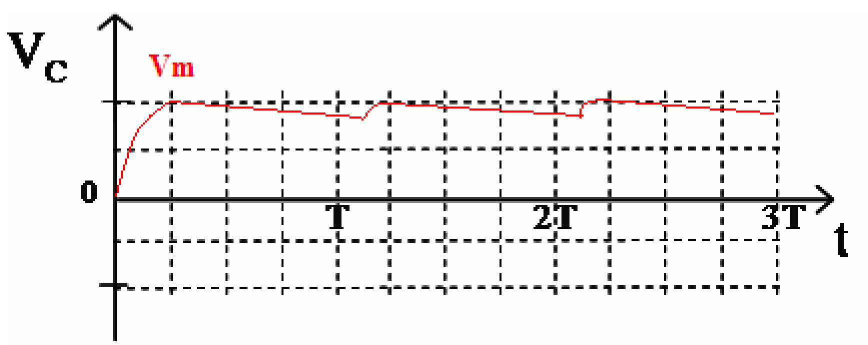


(a) (8 pts) Sketch $V_C(t)$? Label all key values.

The capacitor is initially able to charge up, since V_out starts at 0V and so the diode is a short. However, the capacitor is not able to discharge through the diode since the diode is an open when reverse biased. Thus, the capacitor discharges through the resistor. Since the RC constant is large, we have either:

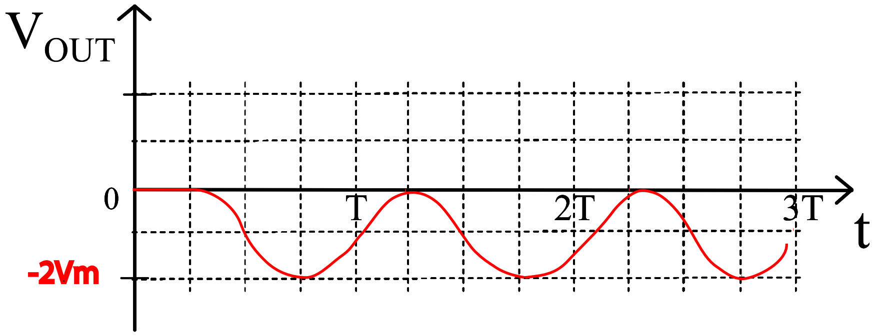


Or:

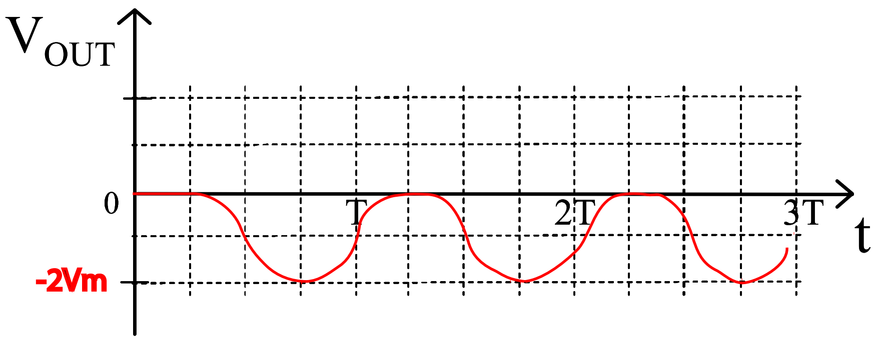


(b) (8 pts) Sketch $V_{OUT}(t)$? Label all key values.

Simple application of KVL gives that V_out = V_in – V_c. The respective sketches of V_out are:



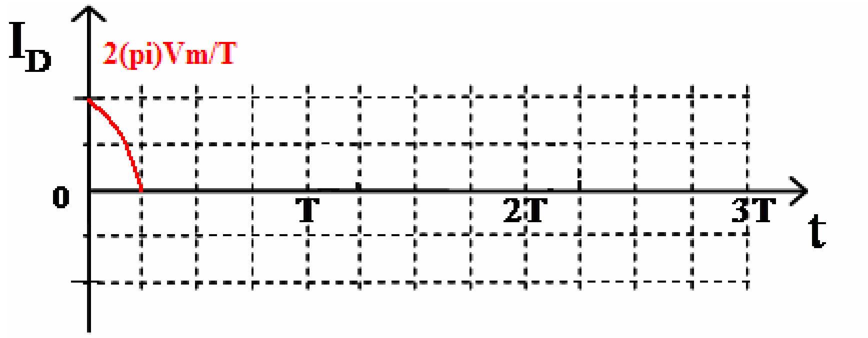
Or:



Note the concavity of the curves above.

(c) (8 pts) Explain what is happening for different time duration.
The capacitor is initially able to charge up, since V_out starts at 0V and so the diode allows current flow in the positive direction. However, the capacitor is not able to discharge through the diode since the diode is an open when reverse biased. Thus, the capacitor discharges through the resistor. Since the RC time constant is large, the capacitor will discharge very slowly (in the limit it will not discharge at all). When V_C matches V_in, then V_out is 0V and so the diode will again allow the capacitor to charge up. We repeat this process.

(d) (6 pts) Sketch $I_D(t)$? Label all key values.



Or: