EECS 40, Fall 2006
Prof. Chang-Hasnain
Midterm #2

October 25, 2006
Total Time Allotted: 50 minutes
Total Points: 100 / Bonus: 10 pts

1. This is a closed book exam. However, you are allowed to bring one page (8.5" x 11"), single-sided notes PLUS your 1-page notes from midterm 1.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. Slide rules are allowed.
4. SHOW all the steps on the exam. **Answers without steps will be given only a small percentage of credits.** Partial credits will be given if you have proper steps but no final answers.
5. **Remember to put down units.** Points will be taken off for answers without units.

Last (Family) Name: Perfect

First Name: Peter

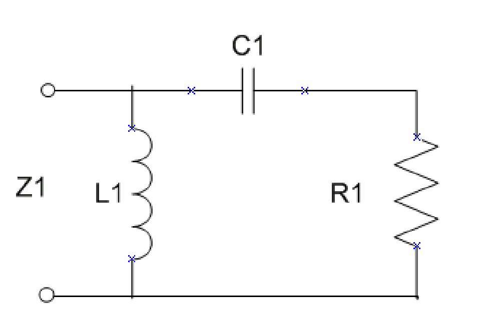
Student ID: 314159265 Discussion Session: 2718

Signature: PP

Score:	**110**
Problem 1 (16 pts) Complex Impedances	16
Problem 2 (54 pts): Bode Plots	54
Bonus (10 pts):	10
Problem 3 (30 pts): Second-order Circuits	30
Total	110

1. [16 points] Parallel and Series Complex Impedance

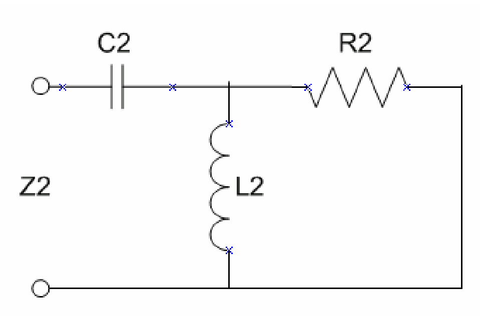
a) [8 pts] What is the complex impedance Z_1?



$$
\begin{aligned}
Z_1 &= Z_L \,//\, (Z_C + R) \\
&= j\omega L \,//\, \left(\frac{1}{j\omega C} + R\right) = j\omega L \,//\, \left(\frac{1 + j\omega RC}{j\omega C}\right) \\
&= \left(\frac{1}{j\omega L} + \frac{j\omega C}{1 + j\omega RC}\right)^{-1} = \left(\frac{1 + j\omega RC + (j\omega L)(j\omega C)}{j\omega L\,(1 + j\omega RC)}\right)^{-1} = \left(\frac{j\omega L\,(1 + j\omega RC)}{1 + j\omega RC + j^2\omega^2 LC}\right) \\
&= \frac{-\omega^2 RLC + j\omega L}{1 - \omega^2 LC + j\omega RC}
\end{aligned}
$$

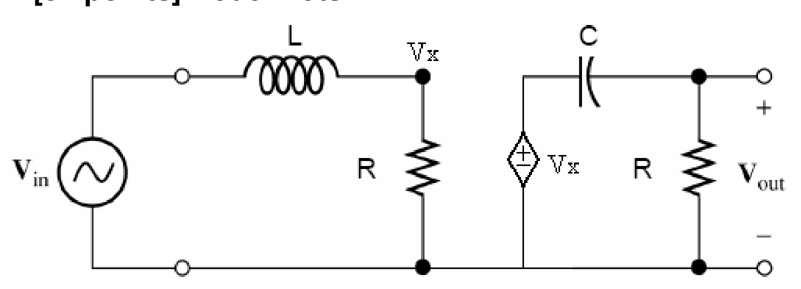
(subscripts omitted for clarity)

b) [8 pts] What is the complex impedance Z_2?



$$
\begin{aligned}
Z_1 &= Z_C + (Z_L \,//\, R) \\
&= \frac{1}{j\omega C} + (j\omega L \,//\, R) = \frac{1}{j\omega C} + \left(\frac{1}{j\omega L} + \frac{1}{R}\right)^{-1} \\
&= \frac{1}{j\omega C} + \left(\frac{j\omega RL}{R + j\omega L}\right) = \left(\frac{R + j\omega L + (j\omega C)(j\omega RL)}{j\omega C\,(R + j\omega L)}\right) = \left(\frac{R + j\omega L + j^2\omega^2 RLC)}{j\omega RC + j^2\omega^2 LC}\right) \\
&= \frac{R - \omega^2 RLC + j\omega L}{-\omega^2 LC + j\omega RC}
\end{aligned}
$$

2. [54 points] Bode Plots:



(a) [10 points] For the above circuit, show $H(f) = \frac{1}{1 + j\frac{f}{f_2}} \times \frac{1}{1 - j\frac{f_1}{f}}$

Express f_1 and f_2 in terms of R, L, C. (Hint: Remember $\omega = 2\pi f$)

Voltage divider on left:

$$V_x = \frac{R}{R + j\omega L} V_{in} = \frac{1}{1 + j\omega\frac{L}{R}} V_{in}$$

Voltage divider on right:

$$V_{out} = \frac{R}{R + \frac{1}{j\omega C}} V_x = \frac{1}{1 - j\frac{1}{\omega RC}} V_x$$

Combining, $V_{out} = \frac{1}{1 - j\frac{1}{\omega RC}} \times \frac{1}{1 + j\omega\frac{L}{R}} V_{in}$

Swapping the terms, $H(f) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega\frac{L}{R}} \times \frac{1}{1 - j\frac{1}{\omega RC}}$

$$= \frac{1}{1 + j2\pi f\frac{L}{R}} \times \frac{1}{1 - j\frac{1}{2\pi fRC}}$$

f_1 *occurs with* f *in the denominator, and* f_2 with f in the numerator:

$$f_1 = \frac{1}{2\pi RC} \qquad f_2 = \frac{R}{2\pi L}$$

(b) [6 points] Now Let R = 1kΩ, L = 0.16 mH, C = 0.16 uF, what are f_1 and f_2? Remember to put down units.

$$f_1 = \frac{1}{2\pi(1\,k\Omega)(0.16\,\mu F)} = \frac{6.25}{2\pi \times 10^{-3}\,s} \approx 1\,kHz$$

$$f_2 = \frac{1\,k\Omega}{2\pi(0.16\,mF)} = \frac{6.25 \times 10^6}{2\pi\ \ s} \approx 1\,MHz$$

(c) [22 pt] Bode Magnitude Plot. ***You must put down all the steps leading to your results.***
Hint: **You may consider** $f_1 \ll f_2$

[4 points] Write down the expression for $y = 10\log\left|H(f)\right|^2$

$$y = 10\log\left|\frac{1}{1+j\frac{f}{f_2}}\right|^2 + 10\log\left|\frac{1}{1-j\frac{f_1}{f}}\right|^2$$
$$= -10\log\left(1+\left(\frac{f}{f_2}\right)^2\right) - 10\log\left(1+\left(\frac{f_1}{f}\right)^2\right)$$

Units are: dB
Note: The other acceptable expression can be found by multiplying the terms in H(f), and finding the magnitude of the resulting product.

[4 points] As frequency goes to a very small value, what is the slope of y as a function of $\log f$?

Constant 1 dominates in left term, f^{-1} *dominates in the right term*:

$$y = -10\log(1) - 10\log\left(\frac{f_1}{f}\right)^2 = 0 + 20\log\left(\frac{f}{f_1}\right)$$
$$= 20\log f - 20\log f_1$$

Slope: 20 dB / decade

[4 points] As frequency goes to a very large value, what is the slope of y as a function of $\log f$?

f *dominates in left term, constant 1 dominates in the right term*:

$$y = -10\log\left(\frac{f}{f_2}\right)^2 - 10\log(1) = -20\log\left(\frac{f}{f_2}\right) - 0$$
$$= -20\log f - (-20\log f_2)$$

Slope: -20 dB / decade

[4 points] What is y, $f_1 \ll f \ll f_2$?
In both terms, constant 1 dominates:

$$y = -10\log(1) - 10\log(1) = -0 - 0$$
$$= 0\ dB$$

[2 points] What is y at f_1?

$$y = -10\log\left(1+\left(\frac{f_1}{f_2}\right)^2\right) - 10\log\left(1+\left(\frac{f_1}{f_1}\right)^2\right)$$

Since $f_1 \ll f_2$, $y = -10\log(1) - 10\log(1+1)$
$$= -10\log 2 = -3\ dB$$

[2 points] What is y at f_2?

$$y = -10\log\left(1+\left(\frac{f_2}{f_2}\right)^2\right) - 10\log\left(1+\left(\frac{f_1}{f_2}\right)^2\right)$$

Again, $f_1 \ll f_2$, $y = -10\log(1+1) - 10\log(1)$

$$= -10\log 2 = -3\ dB$$

[2 points] What filter is this?

Bandpass filter

Bonus [5 points] If the input $|V_{in}| = 1\ V$ and the frequency is 1 MHz, what is the output $|V_{out}|$?

$f_2 = 1\ MHz$**,** *so* $y = -3\ dB = 10\log(\tfrac{1}{2})$

$$\frac{1}{2} = |H(f)|^2 = \left|\frac{V_{out}}{V_{in}}\right|^2 \qquad |V_{out}| = \frac{1}{\sqrt{2}}|V_{in}| \approx 0.707V$$

Bonus [5 points] If the input $|V_{in}| = 1\ V$ and the frequency is 10 MHz, what is the output $|V_{out}|$?

$10\ MHz = 10 f_2$ *: one decade past break frequency*

At large f *, slope is -20 dB/decade. Thus, y = -20 dB, since* f *is 1 decade higher than the break frequency, where the Bode approximation is 0 dB.*

$y = -20\ dB = 10\log(\tfrac{1}{100})$

$$\frac{1}{100} = |H(f)|^2 = \left|\frac{V_{out}}{V_{in}}\right|^2 \qquad |V_{out}| = \frac{1}{\sqrt{100}}|V_{in}| = 0.1V$$

(d) [16 pt total] Bode Phase Plot. ***You must put down all the steps leading to your results. Hint:*** **You may consider** $f_1 \ll f_2$

[4 points] Write down the expression for $\angle H(f)$

$$\angle H(f) = \tan^{-1} 0 - \tan^{-1}\left(\frac{f}{f_2}\right) + \tan^{-1} 0 - \tan^{-1}\left(-\frac{f_1}{f}\right)$$
$$= \tan^{-1}\left(\frac{f_1}{f}\right) - \tan^{-1}\left(\frac{f}{f_2}\right)$$

[4 points] What does the value of $\angle H(f)$ approaches to as $f \to 0$?

$$\angle H(f) = \tan^{-1}(\infty) - \tan^{-1}(0) = \frac{\pi}{2}\ radians$$

[4 points] What does the value of $\angle H(f)$ approaches to as $f \to \infty$?

$$\angle H(f) = \tan^{-1}(0) - \tan^{-1}(\infty) = -\frac{\pi}{2}\ radians$$

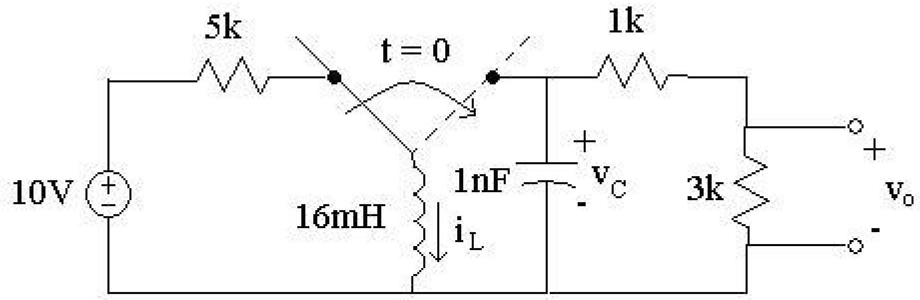
[2 points] What is $\angle H(f)$ at $f = f_1$?

$$\angle H(f) = \tan^{-1}\left(\frac{f_1}{f_1}\right) - \tan^{-1}\left(\frac{f_1}{f_2}\right) = \tan^{-1}(1) - \tan^{-1}(0)$$
$$= \frac{\pi}{4}\ radians$$

[2 points] What is $\angle H(f)$ at $f = f_2$?

$$\angle H(f) = \tan^{-1}\left(\frac{f_1}{f_2}\right) - \tan^{-1}\left(\frac{f_1}{f_2}\right) = \tan^{-1}(0) - \tan^{-1}(1)$$
$$= -\frac{\pi}{4}\ radians$$

3. [30 points] Second-order Circuits:



Assume the switch has been to the left for a long time before switching to the right at t = 0.

(a) Find the following values: [18 points] (Hint: What is $v_o(t)$ in terms of $v_C(t)$?)

$i_L(0+) = \frac{10\,V}{5\,k\Omega} = 2\,mA$ *Inductor is short circuit right before switch is thrown.*	$i_L(\infty) = 0\,A$ *All power is dissipated through resistors.*
$v_C(0+) = 0\,V$ *No stored charge.*	$v_C(\infty) = 0\,V$ *Discharges through resistors.*
$v_o(0+) = 0\,V$ *All current initially flows through capacitor (short).*	$v_o(\infty) = 0\,V$ *Power dissipated, no current.*
$\frac{d}{dt} i_L(0+) = 0\,A/s$ $v_L(0+) = L\frac{d}{dt} i_L(0+) = V_C(0+) = 0\,V$	
$\frac{d}{dt} v_C(0+) = -2\,MV/s$ $i_C(0+) = C\frac{d}{dt} v_C(0+) \quad \Rightarrow \quad \frac{d}{dt} v_C(0+) = \frac{-i_L(0+)}{C} = \frac{-2\,mA}{1\,nF} = -2\times 10^6\,V/s$	
$\frac{d}{dt} v_o(0+) = -1.5\,MV/s$ *Voltage divider:* $v_o = \frac{3\,k\Omega}{1\,k\Omega + 4k\Omega} v_C \Rightarrow \frac{d}{dt} v_o(t) = \frac{3}{4}\frac{d}{dt} v_C(t)$	

(b) [6 points] Write the differential equation in terms of v_c.

KCL @ node above capacitor, (the 2 resistors are combined into R):

$$0 = i_L(t) + C\frac{d}{dt}v_C(t) + \frac{1}{R}v_C(t)$$
$$= \frac{1}{L}\int v_C(t)dt + C\frac{d}{dt}v_C(t) + \frac{1}{R}v_C(t)$$

Differentiating,

$$0 = \frac{1}{L}v_C(t) + C\frac{d^2}{dt}v_C(t) + \frac{1}{R}\frac{d}{dt}v_C(t)$$

$$0 = \frac{d^2}{dt}v_C(t) + \frac{1}{RC}\frac{d}{dt}v_C(t) + \frac{1}{LC}v_C(t)$$

(c) [6 points] What are the values of the natural frequency (ω_0) and the damping ratio (ζ)?

Damped harmonic oscillation:

$$0 = \frac{d^2}{dt}v_C(t) + 2\alpha\frac{d}{dt}v_C(t) + \omega_0^2 v_C(t)$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(16\,mH)(1\,nF)}} = \frac{1}{\sqrt{16\times10^{-12}s^2}}$$
$$= 250\,krad/s$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(4\,k\Omega)(1\,nF)} = \frac{1}{8\times10^{-6}s}$$
$$= 125\,krad/s$$

$$\zeta = \frac{\alpha}{\omega_0} = \frac{125\,krad/s}{250\,krad/s} = 0.5$$ *(underdamped behavior)*