Ö. Savaş NAME: **SOLUTIONS** 9:10-10:00, Friday, October 3, 2008

ME 106 FLUID MECHANICS

EXAM 1 – open book, open notes, no external communication

1.(15+15=30%)

Calculate the pressure difference between the floor and the ceiling of this lecture hall. The ceiling is about 6m high. The room temperature is 15°C. The lecture hall is about 100m above the sea level. What would the difference be if the lecture hall were at Lake Tahoe, where the elevation is about 2000 meters?

Determine the local density using Table C.2 or class notes (Eq. 8)

$$\Delta p = \rho g \Delta z = 1.23\ kg/m^3 \times 9.81\ m/s^2 \times 6\ m = 72\ Pa \quad \text{in Berkeley}$$

$$\Delta p = \rho g \Delta z = 1.00\ kg/m^3 \times 9.81\ m/s^2 \times 6\ m = 59\ Pa \quad \text{at Lake Tahoe}$$

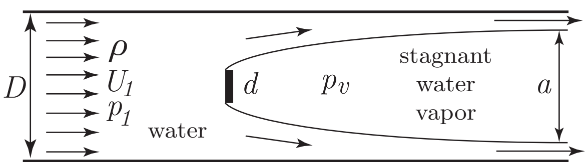
2.(10+5+20=35%)

Consider the flow of water in a channel of diameter D. The velocity is uniform U_1, and pressure is p_1. A disk of diameter d is placed centrally in the channel. When the overall pressure is low enough, a cavitation bubble forms downstream of the disk, in which the pressure is the vapor pressure of water $p_v \ll p_1$. Far downstream, the bubble diameter is a and water flow velocity in the water layer is uniform. Ignore all gravitational and viscous effects.

(a) Determine the water velocity far downstream.

(b) Determine the cavitation bubble diameter a.

(c) Determine the drag force on the disk.



Bernoulli Eq:

$$U_2 = U_1\sqrt{1 + 2(p_1 - p_v)/\rho U_1^2}$$

Mass Conservation: ($A_D = \pi D^2/4,\ A_d = \pi d^2/4,\ A_a = \pi a^2/4$)

$$\dot{m} = \rho U_1 A_D = \rho U_2 (A_D - A_a) \Longrightarrow a = D\sqrt{1 - U_1/U_2}$$

Momentum Conservation:

$$p_1 A_D - \dot{m}U_1 - p_v(A_D - A_d) + \dot{m}U_2 - F_d = 0 \Longrightarrow F_d = (p_1 - p_v)A_D + p_v A_d + \dot{m}(U_2 - U_1)$$

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3. (5+10+15+5=35%)

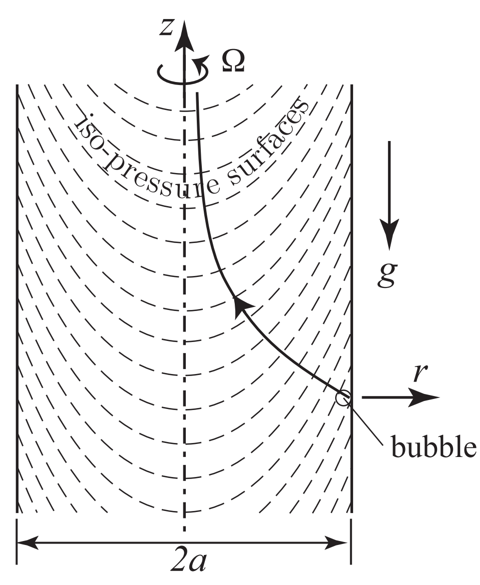
Consider a liquid in solid body rotation in an infinitely long vertical cylindrical container in the gravitational field g. The rotation rate is Ω and the cylinder radius a. Suppose a small air bubble is released from the wall of the cylinder. Since the bubble will follow the line of the steepest pressure drop, it will rise and move toward the axis.

(a) Draw the pressure gradient vector at the location of the bubble.

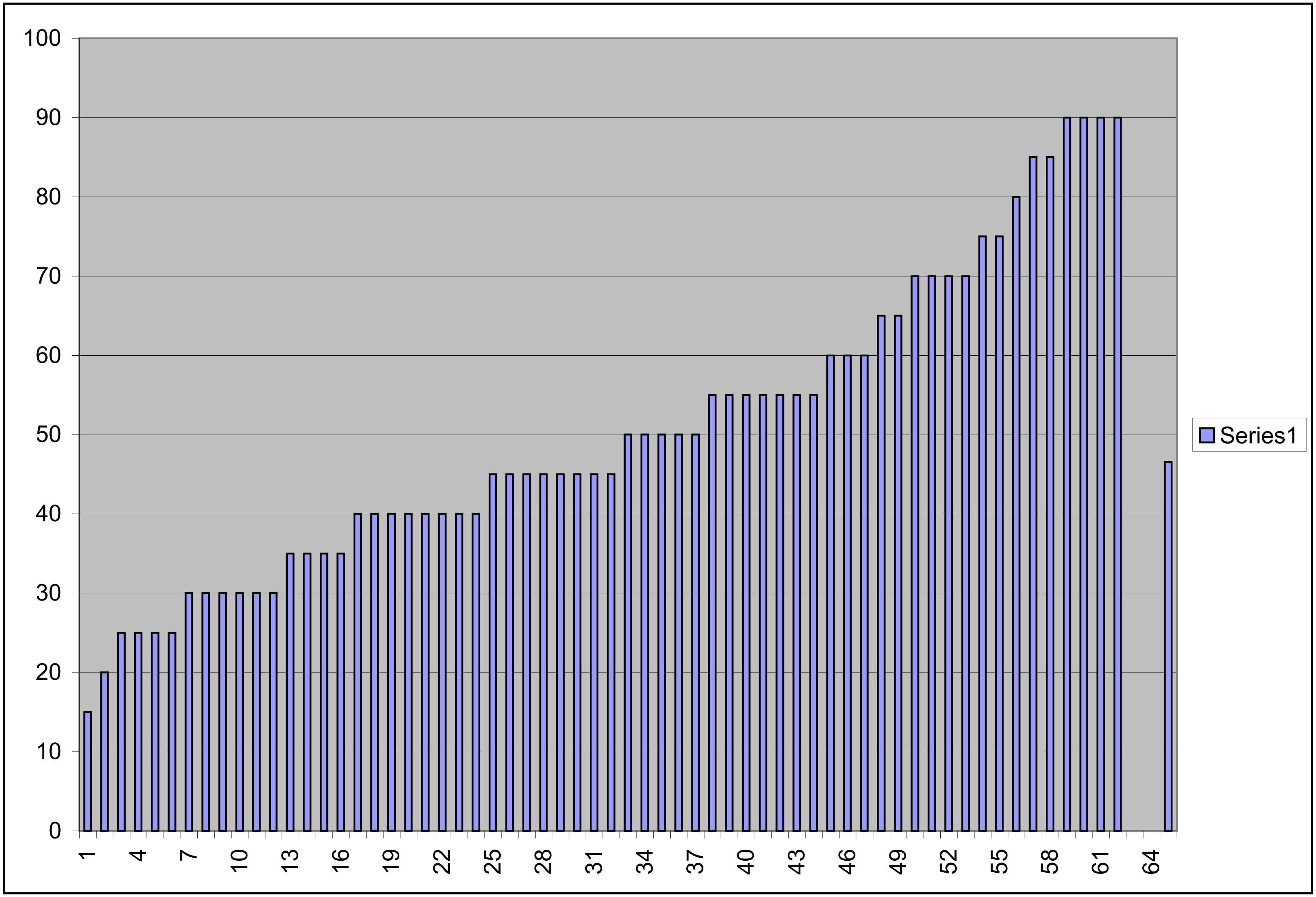
(b) Obtain the equation describing the path of the bubble as observed from the rotating reference frame.

(c) Determine the trajectory of the bubble.

(d) Sketch the trajectory.



$$-d\mathbf{x} \times \nabla p = 0 \Longrightarrow (-dr, 0, -dz) \times (\rho r \Omega^2, 0, -\rho g) = 0 \Longrightarrow \frac{-dr}{r\Omega^2} = \frac{dz}{g} \Longrightarrow r = ae^{-\Omega^2 z/g}$$



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