1. Let the angular position of the rod be measured by θ with respect to the vertical. When the force $F = 12$ lb is applied, the two balls move in a circle relative to G. With respect to the mass center G, the absolute and relative angular momenta are equal. Thus

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G = \dot{\mathbf{H}}'_G$$

Denote by b the distance of G from the 2-lb ball. By moment balance,

$$4(10-b) = 2b \qquad \Rightarrow \qquad b = \frac{20}{3} = 6.67 \text{ in}$$

It follows that

$$\sum M_G = F\left(\frac{6+3.33}{12}\right) = 9.33 \text{ lb-ft-sec} \qquad \circlearrowright$$

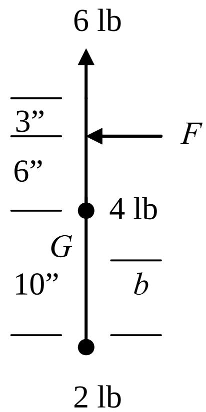
and

$$H'_G = \sum \rho_i (m_i \dot{\rho}_i) = \sum m_i \rho_i^2 \dot{\theta}$$

$$= \frac{4}{32.2}\left(\frac{3.33}{12}\right)^2 \dot{\theta} + \frac{2}{32.2}\left(\frac{6.67}{12}\right)^2 \dot{\theta} = 0.0288\dot{\theta} \text{ lb-ft-sec}$$

Hence,

$$\sum M_G = \dot{H}'_G \quad \Rightarrow \qquad \ddot{\theta} = 325 \text{ rad/sec}^2 \qquad \circlearrowright$$



2. The change in kinetic energy before and after impact is

$$\Delta T = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 - \frac{1}{2}m_1 v_1^2 - \frac{1}{2}m_2 v_2^2$$

$$= \frac{1}{2}m_1(v_1' + v_1)(v_1' - v_1) + \frac{1}{2}m_2(v_2' + v_2)(v_2' - v_2)$$

From the conservation of linear momentum,

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\Rightarrow \qquad m_1(v_1' - v_1) = -m_2(v_2' - v_2)$$

$$\Rightarrow \qquad \Delta T = \frac{1}{2}m_1(v_1' - v_1)[(v_1' + v_1) - (v_2' + v_2)]$$

$$= \frac{1}{2}m_1(v_1' - v_1)(v_1 - v_2)\left[1 - \frac{(v_2' - v_1')}{(v_1 - v_2)}\right]$$

$$= \frac{1}{2} m_1 (v_1' - v_1)(v_1 - v_2)(1 - e)$$

As a result, $\Delta T = 0$ if and only if $e = 1$.

3. Since *B* moves in a circle of radius *l* about the fixed point *A*, both its velocity and acceleration are known. Attach a translating *x*-*y* frame to *B* with the *x*-axis in the direction of *AC*. For the two points *B* and *C* on the rod *BC*,

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B} = \mathbf{a}_B + \boldsymbol{\omega}_{BC} \times (\boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}) + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} \qquad (1)$$

Observe that

$$AB = BC \qquad \Rightarrow \qquad \omega_{BC} = -\omega$$

In addition,

$$\mathbf{a}_B = (\mathbf{a}_B)_n + (\mathbf{a}_B)_t = (-l\omega^2 \cos\theta - l\alpha \sin\theta)\,\mathbf{i} + (-l\omega^2 \sin\theta + l\alpha \cos\theta)\,\mathbf{j}$$

$$\boldsymbol{\omega}_{BC} \times (\boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}) = -l\omega^2 \cos\theta\,\mathbf{i} + l\omega^2 \sin\theta\,\mathbf{j}$$

$$\boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} = l\alpha_{BC} \sin\theta\,\mathbf{i} + l\alpha_{BC} \cos\theta\,\mathbf{j}$$

$$\mathbf{a}_C = a_C\,\mathbf{i}$$

Substitute into (1) and equate coefficients of **j**,

$$\alpha_{BC} = -\alpha$$

which is obvious because $AB = BC$. Equate coefficients of **i**,

$$a_C = -2l\omega^2 \cos\theta - 2l\alpha \sin\theta \qquad \longleftarrow$$