MSE 113 / ME 124 Fall 2008 Midterm 2 solution.

Problem 1

a) Some acceptable answers:

- Micro-cracking
- Bonds breaking
- Creep
- Diffusion
- Polymer crazing
- Grain boundary sliding
- Fiber pull-out

b) Use Frank's Rule:

If $b^2 > b_1^2 + b_2^2$ then the reaction will occur

$b^2 = (a^2/2^2)*(1^2+1^2+0^2) = (1/2)\ a^2$

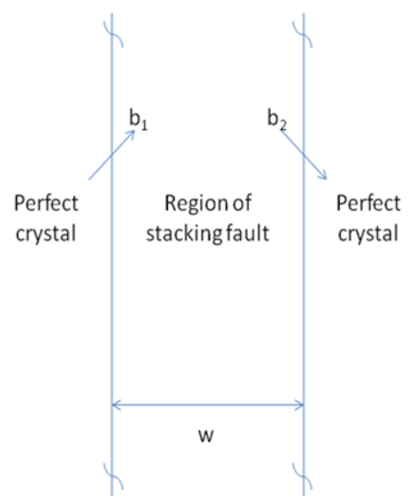
$b_1^2 + b_2^2 = 2\ b_1^2 = 2*(\ a^2/6^2)*(2^2+1^2+1^2) = (1/3)\ a^2$

$(1/2)\ a^2 > (1/3)\ a^2$

<u>The reaction will take place.</u>

c) The dislocation reaction results in a **<u>stacking fault</u>**

d)



The repelling stress on dislocation 1 due to dislocation 2 is

$$\tau = \frac{G\ b_2}{2\ \pi\ w}$$

The repelling force on dislocation 1 is then

$$F = \tau\, b_1 l = \frac{G\, b_2}{2\,\pi\, w} b_1 l$$

Now, if we look at the work done to move dislocation 1 away from dislocation 2 by a distance *dw*, or in other words, we increase w by dw:

$$\text{Work to increase } w \text{ by } \partial w = \gamma[l\ (w + \partial w)] - \gamma\, l\, w$$

The equilibrium separation of the two dislocations will be defined as the width when the work to increase the separation by *dw* equals the work to move a dislocation by *dw*:

$$\gamma[l\ (w + \partial w)] - \gamma\, l\, w = \frac{G\, b_2}{2\,\pi\, w} b_1 l\, \partial w$$

Since $|b_1|=|b_2|$, we'll call it b

$$\gamma\, l\, \partial w = \frac{G\, b^2}{2\,\pi\, w} l\, \partial w$$

$$\boxed{w = \frac{G\, b^2}{2\,\pi\,\gamma}}$$

e) b for the partial dislocations is a/√6

$$w = \frac{G\, b^2}{2\,\pi\,\gamma} = \frac{45 \times 10^9 Pa \cdot (0.368 \times 10^{-9} m)^2}{2 \cdot 6 \cdot \pi \cdot 14 \times 10^{-3} J/m^2} = 1.15 \times 10^{-8} m$$

w = 11.5 nm

which is about 30 times larger than the lattice spacing. This makes sense in the alloy that we have.

Problem 2

a) Some acceptable answers:
   - Toughness, especially at low temperatures
   - Fatigue resistance
   - Low density
   - Oxidation resistance
   - Thermal stability

b) r/t=12, so a thin walled approximation is valid.

Stresses:

$\sigma_{zz} = \sigma_{rr} = \sigma_{r\theta} = \sigma_{\theta z} = \sigma_{rz} = 0$ because the pipe is open ended.

$\sigma_{\theta\theta} = pr/t = p^* [1.5''/(1/8)''] = 12^*p$

Strains:

$\varepsilon_{\theta\theta} = \ln(r/r_0)$

$\varepsilon_{rr} = \ln(t/t_0) = 0$ as $\sigma_{rr} = 0$

Strain Rate:

$$\dot{\varepsilon}_{\theta\theta} = \left(\frac{\sigma_{\theta\theta}}{\sigma_0}\right)^m exp\left[\frac{-H}{kT}\right]$$

*Assume that the lines in the plot are parallel.

We can solve for H at constant stress (use the given line on the plot), or we can solve for it by knowing that the slope of the strain rate vs. 1/T plot is equal to –H/2.303 k:

$$slope = -\frac{H}{2.303\,k}$$

$$H = -slope \cdot 2.303 \cdot k = 37{,}400\,R \cdot 2.303 \cdot 6.79 \times 10^{-23}\frac{inlb}{R} = \mathbf{5.85 \times 10^{-18}\,in\,lb}$$

Solve for m at constant temperature:

$$m = \frac{\Delta\ln(\dot{\varepsilon}_{11})}{\Delta\ln(\sigma_{11})} = \frac{\Delta\log(\dot{\varepsilon}_{11})}{\Delta\log(\sigma_{11})}$$

Or, we can solve for m knowing that the slope of the strain rate vs. stress plot equals m. We are given that the slope=8, so **m=8**

Now, we can find σ_0 by setting both T and σ_{11} constant:

$$\sigma_0 = (\sigma_{11})\left(\frac{\dot{\epsilon_0}}{\dot{\epsilon_{ss}}}\right)^{1/m} e^{-\frac{H}{mkT}}$$

$$\sigma_0 = (\sigma_{11})\left(\frac{1}{\dot{\epsilon_{\theta\theta}}}\right)^{1/m} e^{-\frac{H}{mkT}}$$

$$\sigma_0 = (10^4\,psi)\left(\frac{1}{2\times 10^{-4}}\right)^{1/8} e^{-\frac{5.85\times 10^{-18}\,in\,lb}{8\cdot 6.79\times 10^{-23}\frac{inlb}{R}\cdot 1760\,R}} = \mathbf{63.8\,psi}$$

c) t =60 days = 1440 hours
$\sigma_{\theta\theta}$ = 12p = 12*800 psi = 9600 psi
T = 1250+460 = 1710 R

$$\varepsilon_{\theta\theta} = \dot{\varepsilon}_{\theta\theta}\cdot t$$

$$\dot{\varepsilon}_{\theta\theta,ss} = \left(\frac{\sigma_{\theta\theta}}{\sigma_0}\right)^m exp\left[\frac{-H}{kT}\right]$$

$$\varepsilon_{\theta\theta,ss} = \left(\frac{\sigma_{\theta\theta}}{\sigma_0}\right)^m exp\left[\frac{-H}{kT}\right]\cdot t$$

$$\varepsilon_{\theta\theta,ss} = \left(\frac{9600\,psi}{63.8\,psi}\right)^8 exp\left[\frac{-5.85\times 10^{-18}\,inlb}{6.79\times 10^{-23}\,\frac{inlb}{R}\cdot 1710\,R}\right] hr^{-1}\cdot 1440\,hr = 0.05$$

New r:

$$r = r_0\exp\left[\varepsilon_{total}\right]$$

$$\varepsilon_{total} = \varepsilon_{primary} + \varepsilon_{elastic} + \varepsilon_{ss} + \varepsilon_{tertiary}$$

We will ignore tertiary creep to be conservative.

$$\varepsilon_{total} = 0.005 + \frac{\sigma}{E} + \varepsilon_{\theta\theta,ss}$$

$$r = 1.5\exp[\varepsilon_{total}] = 1.5\exp\left[0.005 + \left(\frac{9{,}600\,psi}{28{,}000{,}000\,psi}\right) + 0.05\right] = 1.585\,inches$$

New t:
$\varepsilon_{rr} = \ln(t/t_0) = 0$, so $t=t_0$.

D_{new} = 1.585*2= 3.17”
t_{new} = 1/8”

Problem 3

a) $\sigma_y = P_y/A = P_y/(0.2m*0.1m) = 50*P_y$

$P_{yeild} = \sigma_Y*A = \sigma_Y/50$

$K_I = 1.12\ \sigma\ \sqrt{(\pi\ a)} = 1.12 * 50 * P\ (\pi * 0.1 \times 10^{-2}\ m)^{1/2}$

$P_{fracture} = K_{IC}\ /(1.12*50*(\pi * 0.1 \times 10^{-2}\ m)^{1/2})$

Material	$P_{Fracture}$ (MN)	P_{Yield} (MN)	Failure mode
High-strength steel	20.7	34	Fracture
Mild steel	63.7	18.8	Yield

Since the load to fracture is lower than the load to yield in the high-strength steel, it will fracture first. Since the load to yield is lower than the load to fracture in the mild steel, it will yield first. The material that yields before it fractures is inherently safer since you can possibly observe some yielding and have time to respond. The high-strength steel would fracture spontaneously at 20.7 MN with no warning. As the weight is 18MN, you would probably be safe assuming that your calculations are correct. The danger is if you are not correct (i.e. this surface flaw is not the worst flaw in the rod), the rod would break with no warning, sending the 18 MN mass at your head. The mild steel has a yield load that is just slightly larger than the hanging load, so it would probably not fail, but if for some reason it did start to go, you would be able to see it yielding and would have some time to react (i.e. run out of the way).

You should go for the challenge, and you should pick the mild steel because it is safer.

b) To determine the validity of using the K approach, we need to know if small-scale yielding applies (r_y << a, W-a):

$$r_y \approx \frac{1}{2\pi}\left(\frac{K_{IC}}{\sigma_y}\right)^2$$

r_y (high-strength steel) = 2×10^{-4} m

r_y (mild steel) = 0.01 m = 1 cm

As we can see, the high-strength steel shows small scale yielding, so we can use the K approach. The mild steel, however, has a plastic zone size on the order of the same dimensions as *a* and *W-a*. This means that the K approach is not really valid. We can proceed to use it, though, if we consider the fact that a plastically deforming material can absorb more energy prior to failure than a perfect linear-elastic material (e.g. consider the area under the stress-strain curve for a ceramic versus a metal). Since the fracture toughness of our material is actually a lot higher than we predicted due to the plasticity, we can use the K approach to find a lower bound on the fracture stress, and therefore, load to fracture.

We also want to know if we are in plane-strain (r_y <<B):

B = 10 cm = 0.1 m

We are definitely in plane strain for high-strength steel, and so K_{IC} is not conservative, it is accurate.

For the mild steel, we are on the cusp of plane-strain (B=10 r_y), and we can't know for sure that the K_C has leveled off completely. As we saw above, we have a lot of plasticity happening in our bar. We are very conservative in using so K_{IC} to predict fracture in the mild steel.