UNIVERSITY OF CALIFORNIA
Electrical Engineering and Computer Sciences

145L MIDTERM #1 (take-home)
September 19, 1994

Due Monday, September 26, 1994

(100 points total, 3 points deducted for each school day late)
(no credit after graded midterms have been returned to students)

PROBLEM 1 (16 points)

For the differential amplifier circuit shown in Course Reader Figure 2.4, and assuming that the open loop gain A is infinite, do the following:

a. Derive the expression for the output V_0 as a function of the four variables R_1, R_2, R_3, and R_4. Your result should be of the form $V_0 = aV_2 - bV_1$.

b. Derive expressions for the differential gain $G_\pm$ and the common mode gain G_c in terms of R_1, R_2, R_3, R_4, using the expression from part a. and the following:

$$V_0 = aV_2 - bV_1 = (a + b)(V_2 - V_1)/2 + (a - b)(V_2 + V_1)/2$$

$$= G_\pm\,(V_2 - V_1) + G_c\,V_c, \text{ where } V_c = (V_1 + V_2)/2$$

c. Using your expressions from step b., derive an expression for the CMRR in terms of R_1, R_2, R_3, R_4.

d. Under what conditions does $G_c = 0$?

e. When $G_c = 0$ is satisfied, what does your expression for $G_\pm$ reduce to?

f. For $G_\pm = 1$ and $G_\pm = 1000$, first set $R_2/R_1 = R_4/R_3 = G_\pm$ and then find the percentage variation in R_3 that causes CMR ≈ 120 dB. Note that $G_\pm$ changes very little. Comment on the resistor accuracy required for a good CMRR at the two differential gains.

Note: $G_\pm$ is primarily determined by R_2/R_1 and R_3 (or R_4) can be used to "fine tune" G_c.

PROBLEM 2 (16 points)

You are given an instrumentation amplifier with a gain that is adjustable from 1 to 1000. At a gain of 1, the bandwidth is 10^6 Hz. At a gain of 1000, the bandwidth is 10^4 Hz.

a. Both input terminals are connected to ground with 5-MΩ resistors. If the input leakage currents on the two inputs are 0.5 nA and 1.5 nA, what is resulting output offset at a gain of 1000? At a gain of 1?

b. What is the output noise in the 10^4-Hz bandwidth at a gain of 1000 due only to the room-temperature Johnson noise in the 5-MΩ resistors? (*Hint:* If two uncorrelated noise sources are added, the rms noises combine as the square root of the sum of their squares.)

c. When the inputs are connected directly to ground, the output-voltage noise with a gain of 1000 is 1 mV rms in the 10^4 Hz bandwidth. When the gain is reduced to

1, the output-voltage noise is 0.1 mV rms in the 10^6 Hz bandwidth. What is the amplifier noise with respect to the input (D_1) and the output (D_0)? (Express the noise in units of nV $Hz^{-1/2}$.)

PROBLEM 3 (10 points)

The classic instrumentation amplifier circuit is shown in figure 2.13 of the course reader (page 82).

Assume the following:

- $R_1 = 100\ \Omega$, $R_2 = 5\ k\Omega$, $R_3 = 1\ k\Omega$, $R_4 = 10\ k\Omega$.
- Input V_+ = +1 volt d.c. plus 1 mV p-p (peak-to-peak) sine wave at 1 kHz
- Input V_- = +1 volt d.c. plus 1 mV p-p sine wave at 1 kHz
- Differential input ($V_+ - V_-$) = 2 mV p-p sine wave at 1 kHz
- Power supply voltages are –10V and + 10V

a. What are the amplitudes of the d.c. and 1 kHz components of V_3?

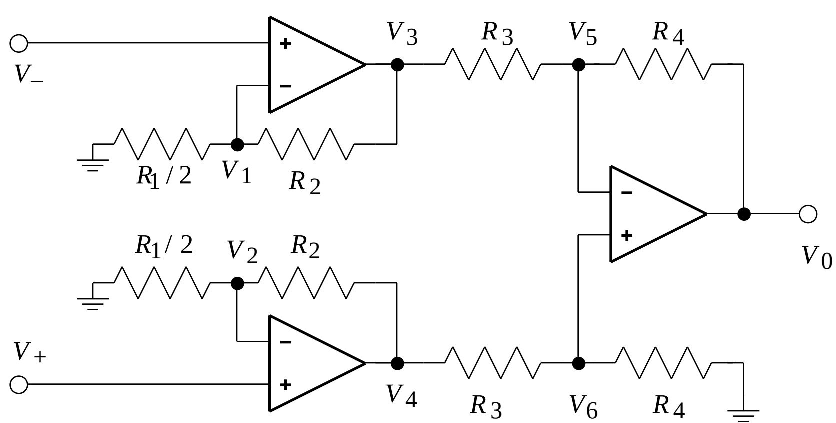
b. What are the amplitudes of the d.c. and 1 kHz components of V_4?

c. What are the amplitudes of the d.c. and 1 kHz components of $V_4 - V_3$?

d. What are the amplitudes of the d.c. and 1 kHz components of V_0?

PROBLEM 4 (16 points)

A new instrumentation amplifier circuit has been proposed, as shown below:



Assume the following (same values as Problem 3):

- $R_1/2 = 50\ \Omega$, $R_2 = 5\ k\Omega$, $R_3 = 1\ k\Omega$, $R_4 = 10\ k\Omega$.
- Input V_+ = +1 volt d.c. plus 1 mV p-p sine wave at 1 kHz
- Input V_- = +1 volt d.c. plus 1 mV p-p sine wave at 1 kHz
- Differential input ($V_+ - V_-$) = 2 mV p-p sine wave at 1 kHz
- Power supply voltages are –10V and + 10V

Answer the following:

a. What are the amplitudes of the d.c. and 1 kHz components of V_3?

b. What are the amplitudes of the d.c. and 1 kHz components of V_4?

c. What are the amplitudes of the d.c. and 1 kHz components of $V_4 - V_3$?

d. What are the amplitudes of the d.c. and 1 kHz components of V_0?

e. Is this circuit design better than the one in Problem 3? Explain your answer.

PROBLEM 5 (10 points)

The formula for the gain of the noninverting amplifier (Course Reader figure 2.3, page 72) is given by:

$$G_{\pm} = \frac{V_0}{V_+ - V_-} = \frac{R_1 + R_2}{R_1}$$

Assume that 10% accuracy resistors are used with values $R_1 = 1$ kΩ, $R_2 = 4$kΩ,

a. what is the gain $G_{\pm}$?

b. what is the accuracy of $G_{\pm}$?

(Hint: use the error propagation formulas given in class and assume that “10% accuracy” means “standard deviation = 10%”)

PROBLEM 6 (16 points)

Design a Butterworth filter that passes frequencies from 0 Hz to 1 kHz with an accuracy of 0.1 dB and rejects frequencies above 10 kHz by a factor of 100 dB.

The *n*th order Butterworth filter has a gain magnitude $|G|$ and phase shift φ given by:

$$|G| = \frac{1}{\sqrt{1 + \left(f/f_c\right)^{2n}}} \qquad \tan\left(\frac{\varphi}{n}\right) = \frac{f}{f_c}$$

a. What is the minimum order n and the corresponding corner frequency f_c that will satisfy the requirements?

b. What are the phase shifts at 100 Hz and 1 kHz?

c. What are the time delays at 100 Hz and 1 kHz associated with those phase shifts?

PROBLEM 7 (16 points)

Design a Butterworth low-pass, four-pole filter using the unity-gain Sallen-Key circuit shown in Figure 2.24 and the filter parameters in Table 2.3. The design constraints are that the –3 dB corner frequency is $f_0 = 1$ kHz, and $R = 10$ kΩ.

a. Determine the component values C_1 and C_2 for each filter section.

b. Sketch the Bode amplitude plot, assuming that the op amp has infinite open-loop gain at all frequencies.

c. What happens when you apply a 100 Hz square wave? Sketch the approximate resulting waveform (amplitude vs. time) and explain.