ME 132, Spring 2003, Final Exam

Name:

# 1	# 2	# 3	# 4	# 5	# 6	# 7
10	20	15	15	15	10	15

# 8	# 9	# 10	# 11	# 12	# 13	TOTAL
15	20	15	15	15	15	180

- Do problems 1-11. Also do problem 12 or problem 13, but not both. Mark above which (of 12 and 13) one you want me to count in your exam.
- Any unmarked summing junctions are positively signed (+).

1. The input u, and output y, of a single-input, single-output system are related by

$$y^{[3]}(t) + 6y^{[2]}(t) + 2y^{[1]}(t) + 3y(t) = 2u^{[2]}(t) - 5u^{[1]}(t) - 5u(t)$$

(a) Find the transfer function from U to Y

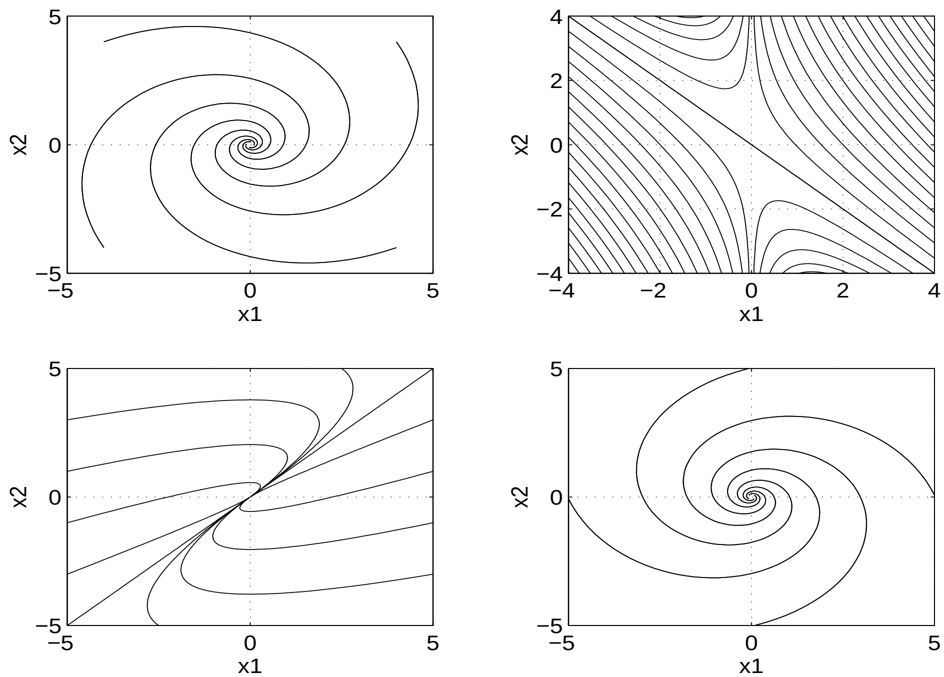
(b) Show that this is a stable system.

(c) If $u(t) \equiv 2$ for all $t \geq 0$, what is the limiting value of y, namely $\lim_{t\to\infty}$?

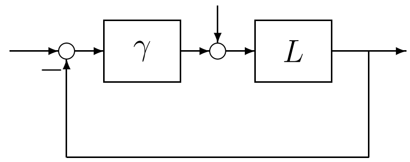
(d) Suppose the input is sinusoidal, $u(t) = \sin(100t)$. In the steady state, what is the approximate amplitude of the sinusoidal output y?

2. Consider the 2-state system governed by the equation $\dot{x}(t) = Ax(t)$. Shown below are the phase-plane plots $(x_1(t) vs. x_2(t))$ for 4 different cases. Match the plots with the A matrices, and correctly draw in arrows indicating the evolution in time. Put your answers on the enlarged graphs included in the solution packet.

$$A_1 = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} -1 & 3 \\ -3 & -1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$$



3. A feedback system is shown below.



Here γ is a constant real number. The open-loop transfer function is $L(s)$, given as

$$L(s) = \frac{30(s+10)(s+10)}{(s+1)(s+1)(s+1)}$$

(a) Determine the characteristic equation for the closed-loop system.

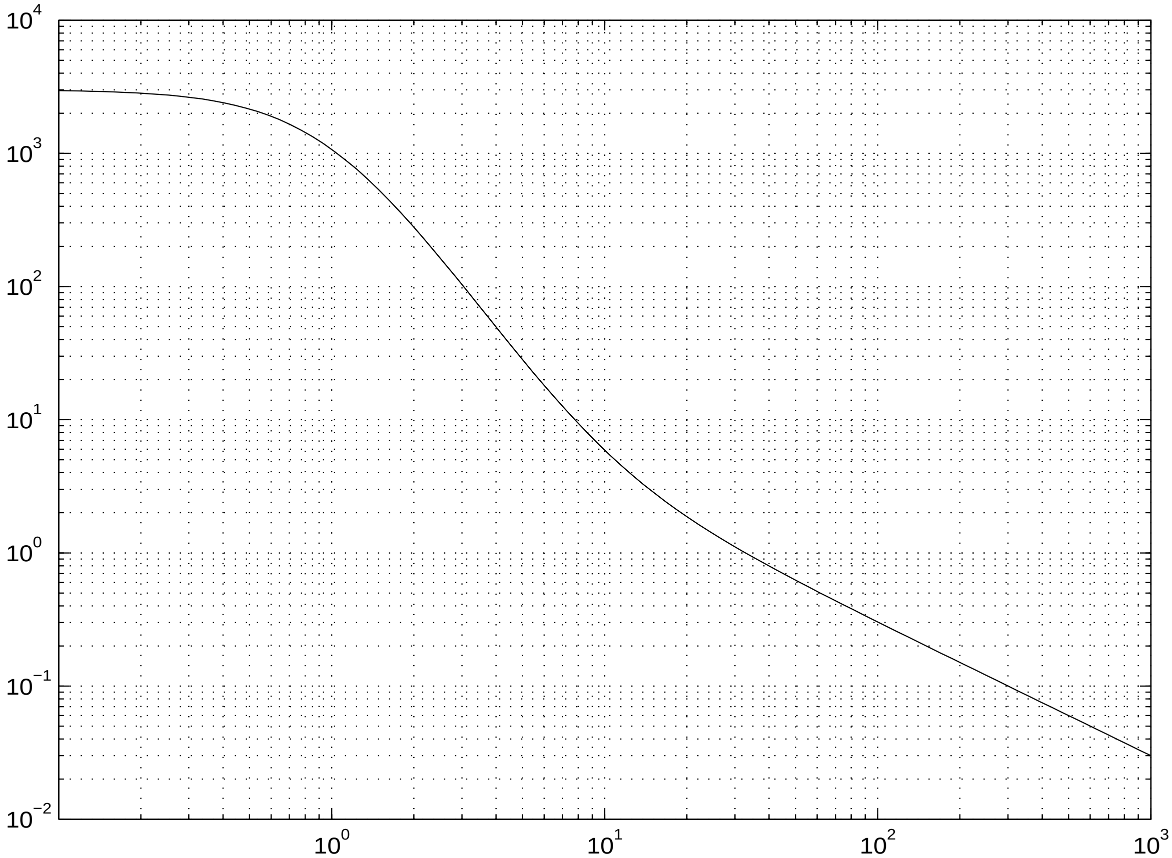
(b) Using the 3rd order test for polynomials, completely determine the range of γ values that result in a stable system. **Here is a hint, provided simply to aid you in checking your answer:** I have already verifed, for instance, that

γ	Closed-loop is
-1	unstable
-0.1	unstable
-0.0002	stable
0.001	stable
0.01	unstable
0.1	stable
1	stable

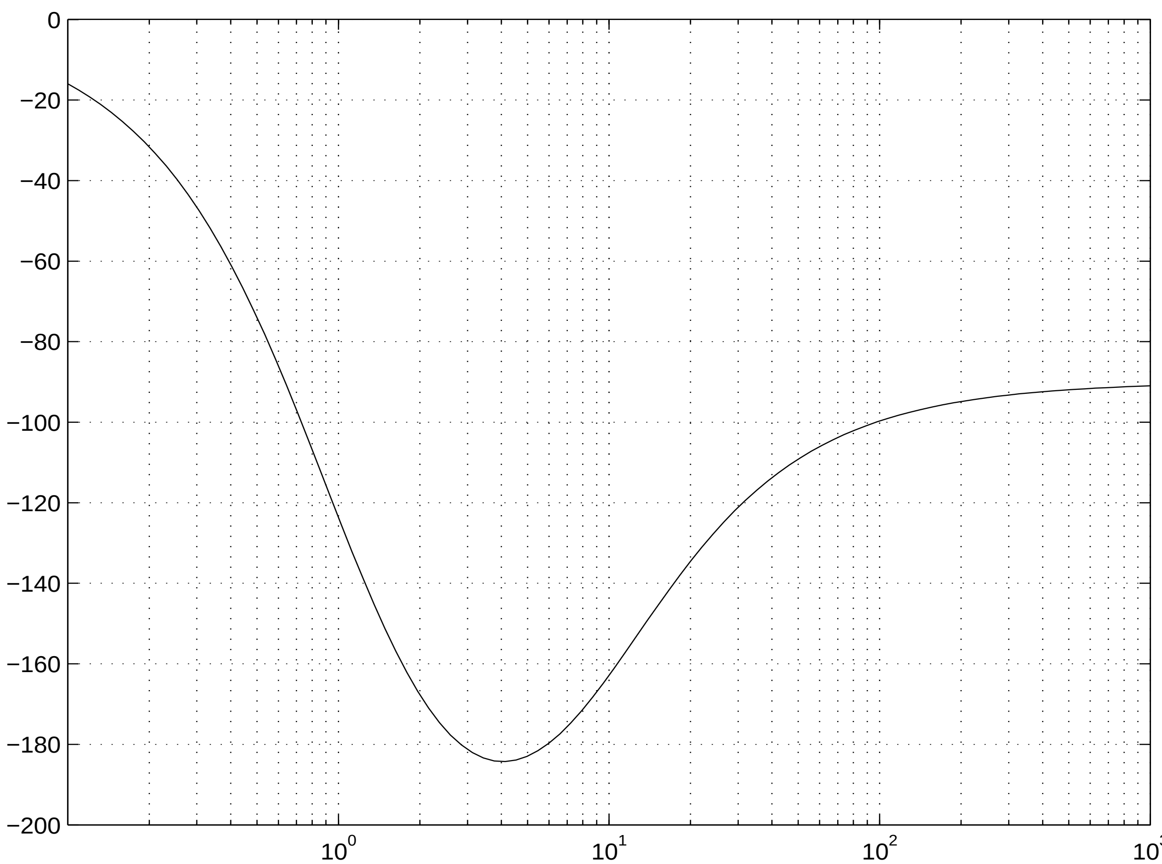
4. In problem 3, you determined the overall possible ranges of γ for which the system shown below is stable. As you determined (and divulged in the **hint**), the stability region includes the point $\gamma = 1$.

   (a) Taking the nominal value of $\gamma = 1$, use the Bode plot of L (provided on the next page) to determine the gain margin of the system. **Show your work. Do not simply take the numbers from problem 3, and express them as a gain margin. Work out the answer independently, using the graph of** L.

   (b) Compare your answer in Problem 3 to the answer here. They are very closely related, but the gain-margin answer has "less" information. Explain.

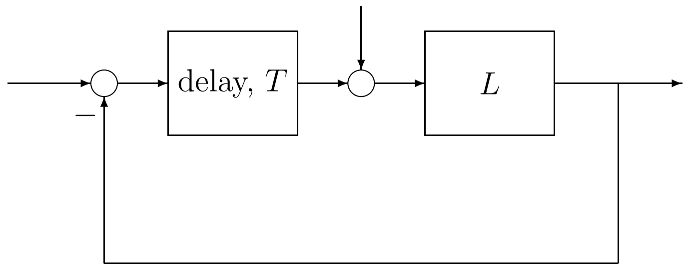


10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^0
10^1
10^2
10^3



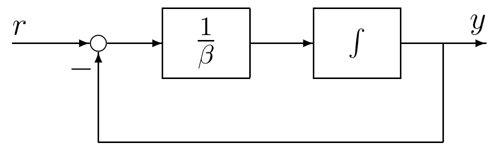
0
−20
−40
−60
−80
−100
−120
−140
−160
−180
−200
10^0
10^1
10^2
10^3

5. Take L from problem 3. There, you determined that for no time delay ($T = 0$), the closed-loop system shown below is stable.



Use the Bode plot of L (provided on the previous page) to determine the time-delay margin of the system.

6. A block diagram is shown below. β is a real number.



(a) What is the differential equation relating y and r?

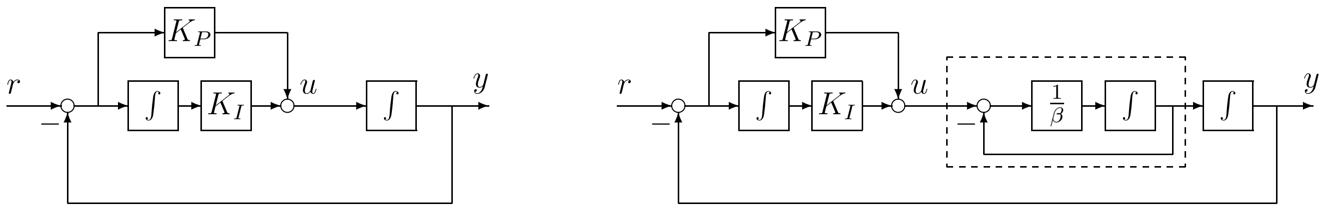
(b) What is the transfer function from R to Y?

(c) Under what conditions (on β) is the system stable?

(d) If the system is stable, what is the time constant?

(e) If the system is stable, what is the steady-state gain from r to y.

7. Shown below are two systems. The system on the left is the **nominal system**, while the system on the right represents a deviation from the nominal (the insertion of the dashed box) and is called the **perturbed system**.



Based on the values of K_P an K_I, and some analysis, you should have a general idea of how the nominal system behaves (eg., the effect of r on u and y). Consider 3 different possibilities (listed below) regarding the relationship between the nominal and perturbed systems:

(a) The perturbed system behaves pretty much the same as the nominal system.

(b) The perturbed system behaves quite differently from the nominal system, but is still stable.

(c) The perturbed system is unstable.

For each row in the table below, which description from above applies? Write **a**, **b**, or **c** in each box. Show work below.

K_P	K_I	β	Your Answer
2.8	4	0.02	
1.4	1	1	
14	100	0.2	
70	2500	0.02	

8. A process, with input v and output y is governed by

$$\dot{y}(t) - 2y(t) = v(t)$$

(a) What is the transfer function from V to Y?

(b) Suppose $y(0) = 1$, and $v(t) \equiv 0$ for all $t \geq 0$. What is the solution $y(t)$ for $t \geq 0$. Is the process stable?

(c) Suppose that the input v is the sum of a control input u and a disturbance input d, so $v(t) = u(t) + d(t)$. Consider a PI control strategy, $u(t) = K_P [r(t) - y(t)] + K_I z(t)$, $\dot{z}(t) = r(t) - y(t)$. Draw a block diagram of the closed-loop system using transfer function representations for the process and the controller. Include the external inputs r and d, and label the signals u and y.

(d) In the closed-loop, what are the transfer functions from R to Y and from D to Y.

(e) In the closed-loop, what are the transfer functions from R to U and from D to U.

(f) For what values of K_P and K_I is the closed-loop system stable?

(g) Choose K_P and K_I so that the closed-loop characteristic equation has roots with $\xi = 0.707(= \frac{1}{\sqrt{2}})$ and $\omega_n = 2$.

9. The pitching-axis of a tail-fin controlled missile is governed by the nonlinear state equations

$$\begin{aligned} \dot{\alpha}(t) &= f(\alpha(t)) \cos \alpha(t) \ + \ q(t) \\ \dot{q}(t) &= h(\alpha(t)) + Eu(t) \end{aligned}$$

Here, the states are $x_1 := \alpha$, the angle-of-attack, and $x_2 := q$, the angular velocity of the pitching axis. The input variable, u, is the deflection of the fin which is mounted at the tail of the missile. E is a physical constant, and $E > 0$. f and h are <u>known</u>, differentiable functions (from wind-tunnel data) of α.

(a) Show that for any specific value of $\bar{\alpha}$, with $|\bar{\alpha}| < \frac{\pi}{2}$, there is a pair $(\bar{q}, \bar{u})$ such that

$$\begin{bmatrix} \bar{\alpha} \\ \bar{q} \end{bmatrix}, \ \bar{u}$$

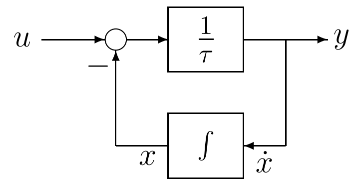
is an equilibrium point of the system (this represents a turn at a constant rate). Your answer should clearly show how $\bar{q}$ and $\bar{u}$ are functions of $\bar{\alpha}$, and will most likely involve the functions f and h.

(b) Calculate the Jacobian Linearization of the missile system about the equilibrium point. In other words, find a 2×2 matrix A, and a 2×1 matrix B, such that while they remain small, the deviation variables $\delta_x(t) := x(t) - \bar{x}, \delta_u(t) := u(t) - \bar{u}$ are approximately governed by

$$\dot{\delta}_x(t) = A\delta_x(t) + B\delta_u(t)$$

Your answers for A and B will be fairly symbolic, and may depend on the derivatives of the functions f and h. Be sure to indicate where the various terms are evaluated.

10. The block diagram below is often called an "approximate differentiator." Note that nowhere in the block diagram is there a differentiating element.



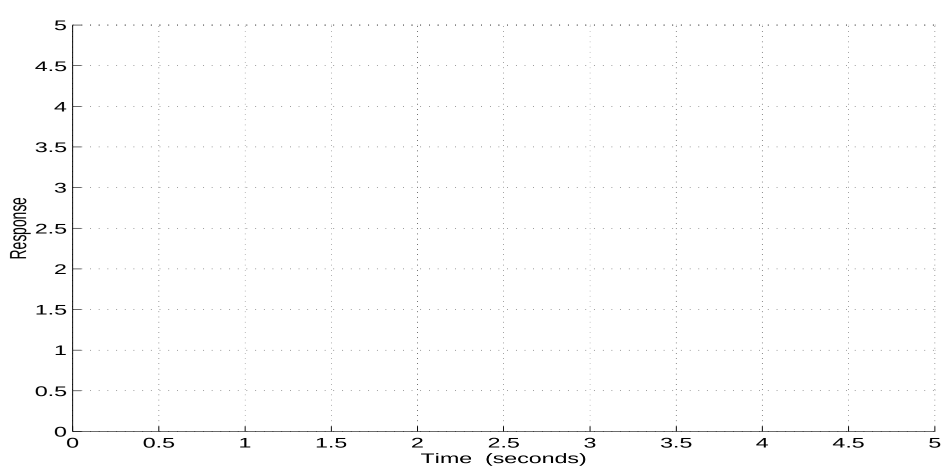
(a) Based on the block diagram, write the differential equation relating x, $\dot{x}$ and u.

(b) Write the equation expressing y in terms of x and u

(c) Show that the transfer function from U to Y is $\frac{s}{\tau s+1}$.

(d) Suppose that the initial condition is $x(0) = 0$. Apply a step input at $t = 0$, so $u(t) = \bar{u}$ for $t > 0$ (here, $\bar{u}$ is just some constant value). Compute the response $x(t)$, for $t \geq 0$.

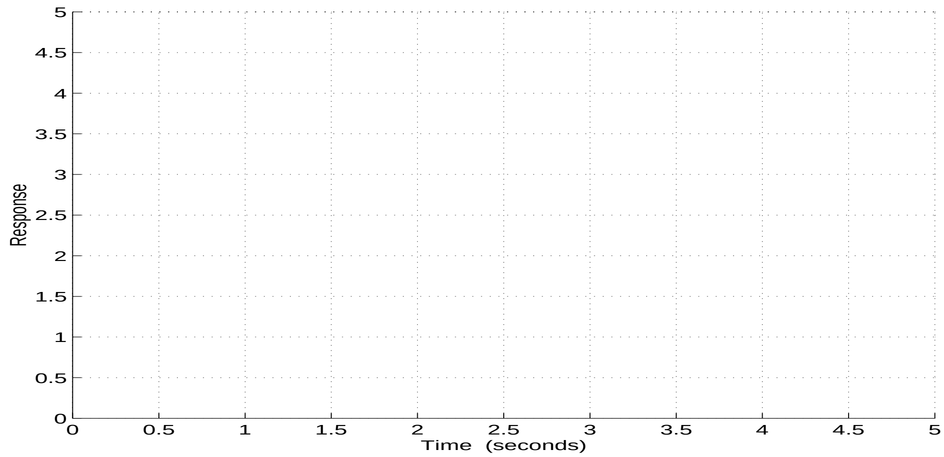
(e) With $x(t)$ computed above, compute the output $y(t)$, and sketch below.



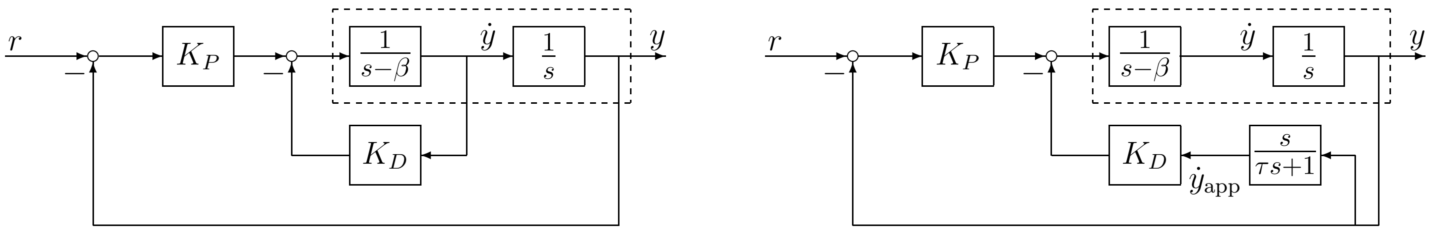
(f) Suppose that the initial condition is $x(0) = 0$, let $\tau = 0.2$. Apply a ramp input (with slope 3)

$$u(t) = 3t \quad \text{for } t \geq 0.$$

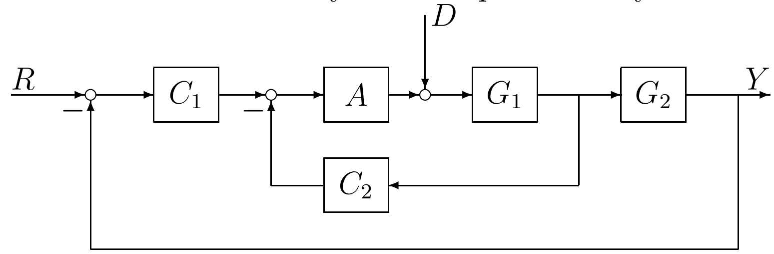
Compute the response $y(t)$, and plot. If you cannot derive the expression for y, guess what it should look like, and plot it below.



11. Block diagrams for two systems are shown below. Two of the blocks are just gains, (K_P and K_D) and the other blocks are described by their transfer functions. The constant β is positive, $\beta > 0$. The system on the left is stable if and only if $K_P > 0$ and $K_D > \beta$ (no need to check this – it is correct). What are the conditions on K_P, K_D and τ, such that the system on the right is stable? **Hint:** Note that τ is the time-constant of the filter in the approximate differentiation used to obtain $\dot{y}_{\text{app}}$ from y. The stability requirements will impose some relationship between it's cutoff frequency $\frac{1}{\tau}$ and the severity (eg., speed) of the unstable dynamics of the process, namely β.

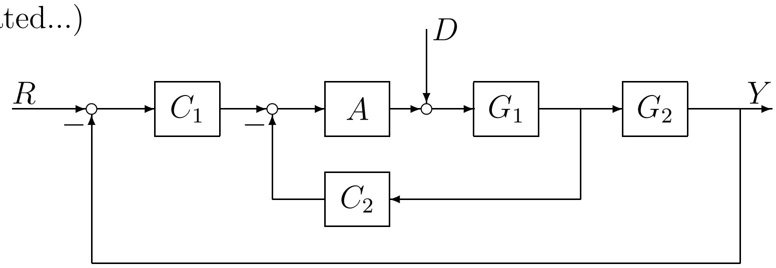


12. A block diagram is shown below. Each system is represented by its transfer function.



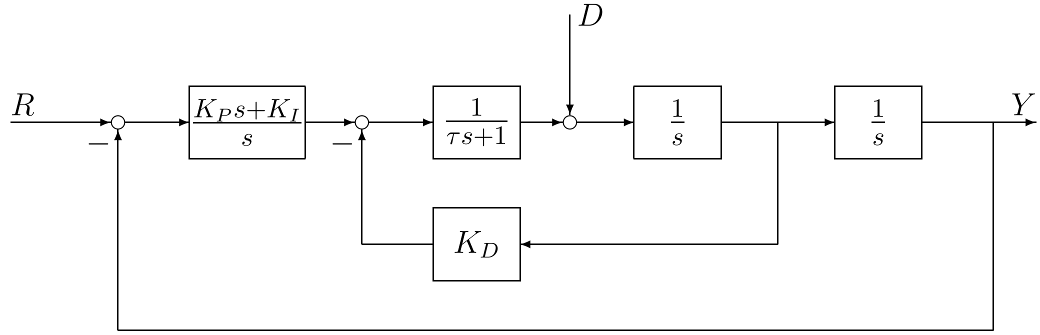
(a) In terms of the transfer functions of the individual blocks, what is the transfer function from R to Y?

(picture repeated...)



(b) In terms of the transfer functions of the individual blocks, what is the transfer function from D to Y?

13. A block diagram is shown below. Each system is represented by its transfer function.



(a) In terms of K_P, K_I, K_D, τ, what is the transfer function from R to Y (hint: the denominator should be 4th order).

(b) In terms of K_P, K_I, K_D, τ, what is the transfer function from D to Y (hint: same as above).

(c) In terms of K_P, K_I, K_D, τ, what is the characteristic polynomial of the closed-loop system?

(d) In terms of K_P, K_I, K_D, τ, what is the differential equation relating r and d to y?