UNIVERSITY OF CALIFORNIA
Electrical Engineering and Computer Sciences

145L MIDTERM (take-home)
Due October 17, 1988

(100 points total, 3 points deducted for each school day late)

QUESTION 1 (8 points):

1.1 Compute the closed loop differential gain G of the differential amplifier of Figure 2.4 of the course reader (page 43) as a function of the ***three variables*** $R_1 = R_3$, $R_2 = R_4$, and the open loop gain A.

1.2 What does your gain expression reduce to in the limit of infinite A?

QUESTION 2 (20 points):

2.1 Compute the output V_0 of the differential amplifier of Figure 2.4 as a function of the ***four variables*** R_1, R_3, R_2, and R_4, assuming that the open loop gain A is infinite. Your result should be of the form $V_0 = aV_2 - bV_1$.

2.2 Compute the differential gain $G_\pm$ and the common mode gain G_c using the following:

$$V_0 = aV_2 - bV_1 = (a + b)(V_2 - V_1)/2 + (a - b)(V_2 + V_1)/2$$
$$= G_\pm (V_2 - V_1) + G_c V_c, \text{ where } V_c = (V_2 + V_1)/2$$

2.3 Compute the CMRR. Comment on the resistor accuracy necessary for a CMR = 120 dB.

2.4 Under what conditions does $G_c = 0$?

2.5 When $G_c = 0$ is satisfied, what does your expression for $G_\pm$ reduce to?

QUESTION 3 (12 points)

Do problem 2.4 of the course reader (page 70).

QUESTION 4 (20 points)

You have a force transducer that uses four metal strain gauges in opposing pairs as shown in Figure 4.14 of the course reader (page 157). You build the bridge circuit shown in the right half of Figure 4.15 (page 158). Assume that the bridge supply is 0.5 Volts, the gauge factor G is 2, and the strains both at the top of the bar (tension) and the bottom of the bar (compression) are $\Delta L/L = 0.1\%$.

4.1 Compute the value of the bridge output $V_0 = V_+ - V_-$.

4.2 Compute the common mode $V_c = (V_+ + V_-)/2$

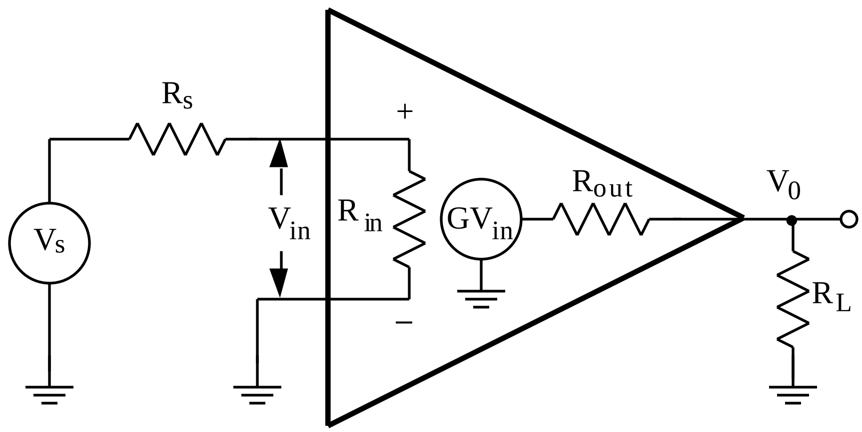
4.3 If you record V_0 using an instrumentation amplifier with a differential gain of 1000, what common mode rejection ratio do you need so that $G_c V_c$ corresponds to $\Delta L/L < 10^{-5}$?

4.4 What noise specification is needed for the instrumentation amplifier if the noise in $\Delta L/L$ is 10^{-6} at 100 Hz?

4.5 If the strain gauges have a resistance that changes with temperature according to $\Delta R/R = k\,(T - T_0)$ where k = 0.1% per °C, what is the effect of a uniform 10°C temperature change on $V_0 = V_+ - V_-$?

QUESTION 5 (20 POINTS)

A power amplifier with a gain $V_0 = G\,V_1$ can be described by the equivalent circuit shown below:



5.1 What is V_0 in terms of V_{in}?

5.2 What is V_{in} in terms of V_s?

5.3 What is V_0 in terms of V_s?

5.4 You want to use this circuit to amplify 1 mV signals from a magnetic tape head (output impedance 1 MΩ) and drive a speaker (input impedance 8 Ω) at 10 Volt amplitude. What are the requirements on R_{in} and R_{out} so that V_{in} is within 1% of V_s and V_0 is within 1% of GV_{in}?

5.5 Comment on the design requirements for R_{in} and R_{out} necessary for specific applications.

QUESTION 6 (20 points)

Design a Butterworth filter that passes frequencies from 0 Hz to 1 kHz with an accuracy of 0.1 dB and rejects frequencies above 10 kHz by a factor of 100 dB.

The nth order Butterworth filter has a gain magnitude |G| and phase shift φ given by:

$$|G| = \frac{1}{\sqrt{1 + \left(f/f_c\right)^{2n}}} \qquad \tan\left(\frac{\varphi}{n}\right) = \frac{f}{f_c}$$

6.1 What is the minimum order n and the corresponding corner frequency f_c that will satisfy the requirements?

6.2 What are the phase shifts at 100 Hz and 1 kHz?

6.3 What are the time delays at 100 Hz and 1 kHz associated with those phase shifts?

6.4 What can you say about the ability to preserve the shape of a 100 Hz square wave? Consider both the effects of the filter on amplitude and phase.

The 100 Hz square wave may be represented by the Fourier series:

$$V(t) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \cos\left(2\pi n f_0 t\right)$$

6.5 The Bessel filter has a phase shift that is proportional to frequency. How would this filter preserve the shape of the 100 Hz square wave?

Equations, some of which you may need:

$$\frac{V_1}{V_1 + V_2} = \frac{R_1}{R_1 + R_2}$$

$$R(T) = R(T_0)\exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

$$T = T_2 + (T_2 - T_1)\, e^{-t/\tau}$$

$$I = I_0\, e^{-kLC}$$

$$I_{rms} = \sqrt{2qI(F_2 - F_1)}$$

$q = 1.60 \times 10^{-19}$ Coulombs

$$V_{rms} = \sqrt{4kTR(F_2 - F_1)}$$

$k = 1.38 \times 10^{-23}$ Volt^2 sec ohm^{-1} °K^{-1}

$$\frac{\Delta R}{R} = G\frac{\Delta L}{L}$$

$$V_0 = G_{\pm}(V_+ - V_-) + G_c(V_+ + V_-)/2$$

"CMRR" $= \dfrac{G_{\pm}}{G_c}$ "CMR" $= 20 \log_{10}\left(\dfrac{G_{\pm}}{G_c}\right)$