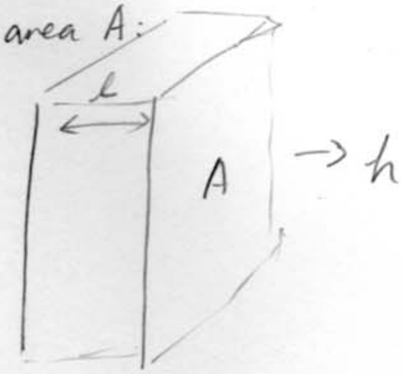
Final exam of 2004.,

1.

(i) Consider a slab with width $l$ and surface area $A$:




The external thermal resistance is:

$$R_{ext} = \frac{1}{hA}$$

The internal thermal resistance is:

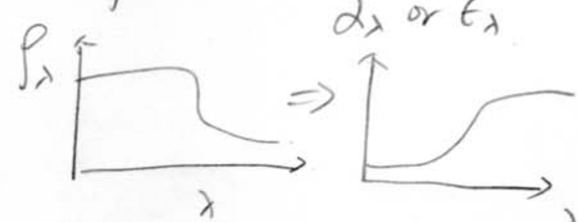
$$R_{int} = \frac{L}{kA}$$

So: $$\frac{R_{int}}{R_{ext}} = \frac{L/kA}{1/hA} = \underline{\frac{hL}{k}} \; \underline{\equiv Bi}$$

(ii). The total-hemispherical emissivity <u>decreases</u> as temperature increases.

Because: based on the wien's disp law:

$$\lambda_{max} T = Const$$




As temperature increases, $\lambda_{max}$ decreases,

but the spectral emissivity is smaller for smaller $\lambda$, so the total [illegible]

( The surface is diffuse opaque: $\epsilon_\lambda = \alpha_\lambda = 1 - \rho_\lambda$ )

Therefore the total-hemispherical emissivity decreases.

( $T \sim 6000K$ )

(iii). False:

The 'potential' or 'voltage' in the radiative resistance network is the emissive power ( $E_b = \sigma T^4$ ) and radiosity ( $J$ );

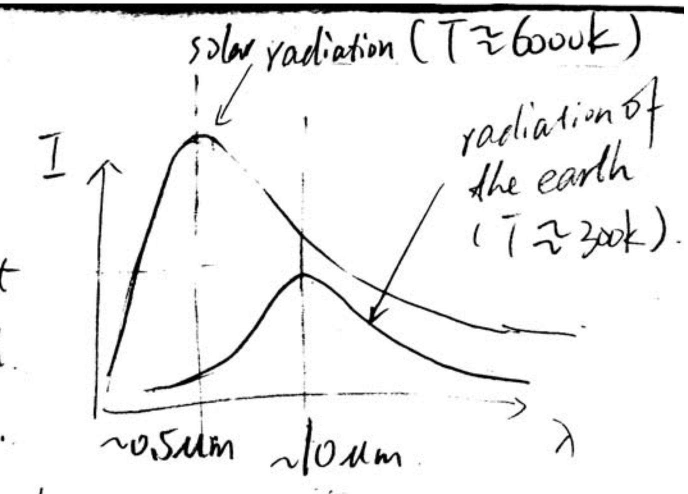
While the 'potential' in the conduction resistance network is the temperature ( $T$ ), which has different physical meaning and unit from $E_b$ or $J$.

Also, the physical meaning of the 'thermal resistance' is different in the two cases: in radiation, $R \left( = \frac{1-\epsilon_1}{\epsilon_1 A_1} \text{ or } \frac{1}{A_1 F_{12}} \right)$ has unit of $[1/m^2]$

while in conduction, $R \left( = \frac{l}{kA} \right)$ has unit of $[K/W]$.

Therefore, the two networks can NOT be combined.

(iv) The first one.

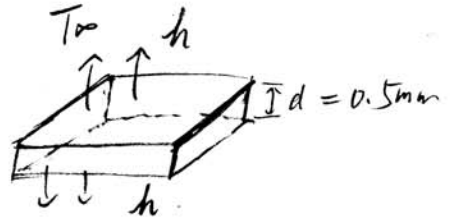



T of the sun is $\sim 6000K$, while T of the earth is $\sim 300K$. So the dominant wavelength ($\lambda_{max}$) of the sun and the earth is $\sim 0.5 \mu m$ and $\sim 10 \mu m$, respectively.

The spectral transmissivity of the first graph has higher $\tau_\lambda$ for $\lambda < 3 \mu m$, which means more solar radiation can transmit through the atmosphere while less radiation of the earth can go through the atmosphere. This results in a warmer earth.

(5) When the velocity of the liquid increases, the macroscopic directional movement of the fluid molecules will play a role in transfering heat between the solid surface to the liquid. While in the pure conduction case, heat is only transfered by the random movement (diffusion) of the molecules, which do not have a preferential direction. Therefore, convection increases the heat transfer rate.

2:




(i): Biot #: $Bi = \frac{hd}{k} = \frac{500 \times 0.5 \times 10^{-3}}{150} = 1.67 \times 10^{-3} < 0.1$

so we can use the lumped capacitance method.

Based on the 1st laws take the whole chip as the C.V.

$$\frac{dE_{cv}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \qquad (1)$$

$$\rho C_p V \frac{dT}{dt} = h2A(T_\infty - T) + 0 + P_0$$

$$\Rightarrow \rho C_p A d \frac{dT}{dt} = 2hA(T_\infty - T) + P_0 = -2hA\left(T - T_\infty - \frac{P_0}{2hA}\right)$$

Let $\theta \equiv T - T_\infty - \frac{P_0}{2hA} = T - T_\infty - \frac{100}{2 \times 500 \times 4 \times 10^{-4}} = T - T_\infty - 250$

$$\boxed{\frac{d\theta}{dt} = -\frac{2h}{\rho C_p d}\theta} \qquad (2)$$

Initial condition:

$$\boxed{t = 0.\ T = T_\infty \text{ or } \theta_0 = -250\ [^\circ C]}$$

(ii) Solve for (2): $\theta = \theta_0 \exp\left(-\frac{t}{\tau}\right) = -250 \exp\left(-\frac{t}{\tau}\right)$

Where $\tau = \frac{\rho C_p d}{2h} = \frac{2300 \times 700 \times 0.5 \times 10^{-3}}{2 \times 500} = 0.805\ [S]$

So: $T = \theta + T_\infty + 250 = T_\infty + 250\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)$

$= T_\infty + 250\left[1 - \exp\left(-\frac{t}{0.805}\right)\right]$

(iii) steady state temperature:

$$t \to \infty \quad T_{ss} = T_\infty + 250$$

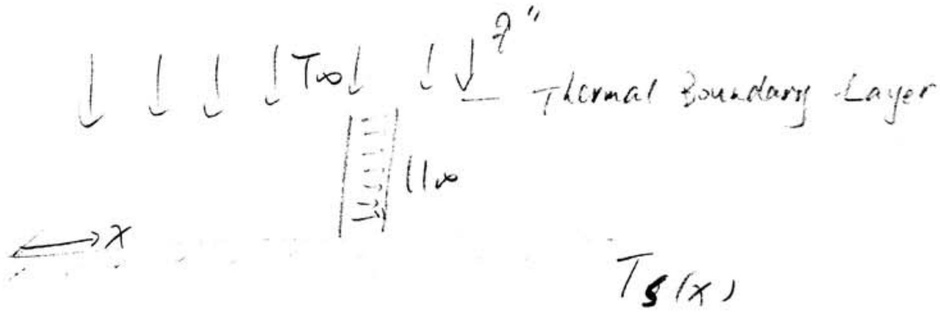
$$\text{or } \theta_{ss} = T_{ss} - T_\infty = 250 \text{ [°C]}$$

or: At S.S: $\dot{E}_{in} + \dot{E}_g = 0$

$$\Rightarrow -2hA(T_{ss} - T_\infty) + P_0 = 0$$

$$\theta_{ss} = T_{ss} - T_\infty = \frac{P_0}{2hA} = 250 \text{ [°C]}$$

Problem 3:




First let's assume a temperature profile for the flow inside the thermal boundary layer:

Define $\theta = \dfrac{T - T_s(x)}{T_\infty - T_s(x)}$ (Note: $T_s$ is unknown) $\left(\eta = \dfrac{y}{\delta_t}\right)$ ①

and assume $\theta = A + B\left(\frac{y}{\delta_t}\right) + C\left(\frac{y}{\delta_t}\right)^2 + D\left(\frac{y}{\delta_t}\right)^3 = A + B\eta + C\eta^2 + D\eta^3$

where $\delta_t$ is the thermal boundary thickness.

Boundary Conditions:

$\eta = 0$: $\theta = 0$,

$$-k\left.\frac{dT}{dy}\right|_{y=0} = -q'' \Rightarrow -k\frac{T_\infty - T_s}{\delta_t} \cdot \left.\frac{d\theta}{d\eta}\right|_{\eta=0} = +q$$

$$\left.\frac{\partial^2 T}{\partial y^2}\right|_{y=0} = 0 \Rightarrow \left.\frac{\partial^2 \theta}{\partial \eta^2}\right|_{\eta=0} = 0$$

$\eta = 1$: $\theta = 1$ and $\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=1} = 0$

So the T profile is:

$$\theta = \frac{T - T_s(x)}{T_\infty - T_s(x)} = \frac{3}{2}\eta - \frac{1}{2}\eta^3 \quad \text{——— (2)}$$

and $\quad -k\,\dfrac{T_\infty - T_s}{\delta_t}\cdot\dfrac{3}{2} = +q''$ (from the B.C: $-k\dfrac{dT}{dy}\Big|_{y=0} = +q''$)

$$\Rightarrow\ T_\infty - T_s = -\frac{2}{3}\, q''\delta_t/k$$

$$T_s = T_\infty + \frac{2}{3}\,\frac{q''\delta_t(x)}{k} \quad \text{——— (3)} \quad \text{(Note: } q'' \text{ is a constant)}$$

Now solve for $\delta_t(x)$

Integral energy equation: $\quad \dfrac{d}{dx}\displaystyle\int_0^{\delta_t} u(T_\infty - T)\,dy = \alpha\frac{\partial T}{\partial y}\Big|_{y=0}$ ——— (4)

Where $\displaystyle\int_0^{\delta_t} u(T_\infty - T)\,dy = \int_0^{\delta_t} u_\infty\cdot[T_\infty - T_s + T_s - T]\,\delta_t\, d\eta$ ——— (5)

$$= u_\infty(T_\infty - T_s)\delta_t \int_0^1 (1-\theta)\,d\eta$$

$$= +\frac{3}{8}u_\infty(T_\infty - T_s)\delta_t$$

$$= +\frac{3}{8}u_\infty\left(\frac{-2}{3}q''\cdot\delta_t/k\right)\cdot\delta_t$$

$$= -\frac{1}{4}u_\infty q''\delta_t^2/k$$

and $\quad \alpha\dfrac{\partial T}{\partial y}\Big|_{y=0} = -\dfrac{\alpha}{k}\left(-k\dfrac{\partial T}{\partial y}\Big|_{y=0}\right) = -\dfrac{\alpha}{k}(+q'') = \dfrac{-\alpha q''}{k}$ ———

Substitute eq (5) and (6) into (4):

$$\frac{d}{dx}\left(-\frac{1}{4}u_\infty q''\delta_t^2/k\right) = \frac{-\alpha q''}{k}$$

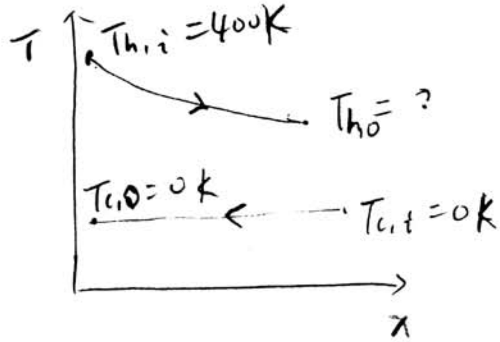
$$\Rightarrow\ \frac{1}{4}u_\infty\frac{d\delta_t^2}{dx} = \alpha$$

$$\Rightarrow\ \delta_t^2 = \frac{4\alpha}{u_\infty}x \ \Rightarrow\ \delta_t = 2\sqrt{\frac{\alpha x}{u_\infty}} \quad \text{——— (7)}$$

From eq (3):

So: $\quad T_s(x) = T_\infty + \dfrac{4}{3}\dfrac{q''}{k}\sqrt{\dfrac{\alpha x}{u_\infty}}$ ——— (8)

Problem #4:




Counter-flow heat exchanger

(2): Initially, the temperature of the ball is at 400K. (Inlet of heat exchanger)

Effective heat transfer coefficient for radiation:

$$h_{rad} = \frac{A \cdot \sigma (T_h^4 - T_\infty^4)}{A(T_h - T_\infty)} = \sigma(T_h^2 + T_\infty^2)(T_h + T_\infty) = \sigma T_h^3$$

$T_h$ is the temperature of the Al ball: $0K < T_h \le 400K$

i.e: $T_h \le 400K$ : $h_{rad} \le \sigma \cdot (400K)^3 = 3.6288 \left[\frac{W}{m^2 K}\right]$

Biot #: $Bi_{rad} = \frac{hL}{k} \le \frac{3.6288 \times (\frac{1}{3} \times 0.1 \times 10^{-3})}{237 \left[\frac{W}{m\text{-}K}\right]} = 5.1 \times 10^{-7}$

So: $Bi_{rad} < 0.1$, we can use the Lumped capacitance method.

$$\rho V C \frac{dT}{dt} = -\sigma A (T^4 - T_\infty^4) = -\sigma A T^4$$

$$\Rightarrow \rho C \frac{1}{6} \pi D^3 \frac{dT}{dt} = -\sigma \pi D^2 T^4$$

$$\frac{dT}{dt} = \frac{-6\sigma T^4}{\rho C D} \quad \text{and } T_i = 400K \ @ \ t = 0$$

$$\Rightarrow -3^{-1} T^{-3} = -\frac{6\sigma}{\rho C D} t - 3^{-1} \cdot T_i^{-3}$$

$$\Rightarrow T^{-3} = \frac{18\sigma}{\rho C D} t + T_i^{-3}$$

$$\Rightarrow T = \left(\frac{18\sigma}{\rho C D} t + T_i^{-3}\right)^{-1/3} = \left(\frac{18 \times 5.67 \times 10^{-8} t}{2 \times 10^6 \times 0.1 \times 10^{-3}} + 400^{-3}\right)^{-1/3}$$

$$= \left(5.103 \times 10^{-9} t + 1.5625 \times 10^{-8}\right)^{-1/3}$$

exit of the

(ii): At the exit of the heat exchange

$$t = \frac{L}{v} = \frac{1\,m}{0.01\,[m/s]} = 100\,[s]$$

$$\Rightarrow T(100s) = \left[5.103\times10^{-9}\times100 + 1.5625\times10^{-8}\right]^{-1/3}\ [K]$$

$$= \underline{123.9\ [K]}$$

(iii) Energy transfered by a single ball:

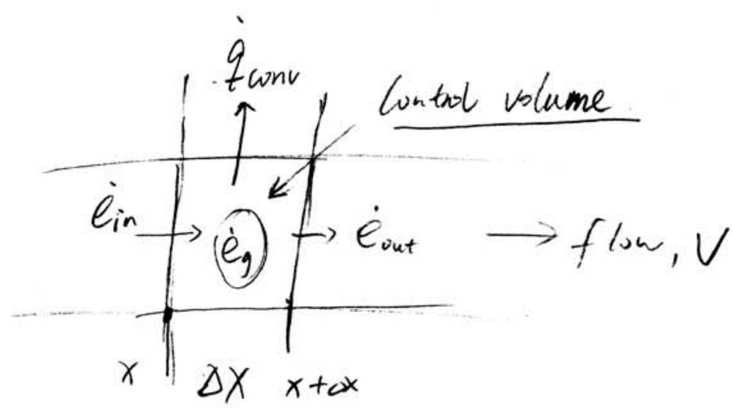
$$q = \rho c\cdot V\,(T_{h,i} - T_{h,o}) = 2\times10^{6}\times\frac{1}{6}\pi\times(0.1\times10^{-3})^{3}\cdot(400-123.9)$$

$$= \underline{2.89\times10^{-4}\,[J]}$$

(iv) $$\varepsilon = \frac{q}{q_{max}} = \frac{C_{min}\,(T_{h,i} - T_{h,o})}{C_{min}\cdot(T_{h,i} - T_{c,i})} = \frac{400-123.9}{400-0} = \underline{0.69}$$

Prob 5:

(i):




$$1^{st}\text{ law}: \quad \dot{e}_{in} + \dot{e}_g - (\dot{e}_{out} + \dot{q}_{conv}) = 0$$

$$\dot{m}c_p\cdot T_m(x) + \dot{q}A_c\Delta x - \left[\dot{m}c_p T_m(x+\Delta x) + hP\Delta x\,(T_m - T_s)\right] = 0$$

$$\Rightarrow \dot{m}c_p\left[T_m(x+\Delta x) - T_m(x)\right] = \dot{q}A_c\,\Delta x - hP\Delta x\,(T_m - T_s)$$

$$\Rightarrow \dot{m}c_p\,\frac{dT_m}{dx} = \dot{q}A_c - hP(T_m - T_s)$$

So: the variation of the mean fluid temperature $T_m$ is:

$$\frac{dT_m}{dx} = \frac{1}{\dot{m}c_p}\left[\dot{q}\frac{\pi D^2}{4} - h\pi D(T_m - T_s)\right]$$

however, as the problem states, $T_m$ remains unchanged along the x-direction, i.e.: $T_m(x) = T_{m,i}$. so:

$$\frac{dT_m}{dx} = \frac{1}{\dot{m}c_p}\left[\dot{q}\frac{\pi D^2}{4} - h\pi D(T_{mi} - T_s)\right] \quad \text{——— ①}$$

(ii) $T_m$ is a constant along x-direction $\Rightarrow \frac{dT_m}{dx} = 0$

so, based on Eq ①: $\frac{1}{\dot{m}c_p}\left[\dot{q}\frac{\pi D^2}{4} - h\pi D(T_{mi} - T_s)\right] = 0$

$$\Rightarrow T_s = T_{mi} - \frac{\dot{q}\pi D^2/4}{h\pi D} = T_{mi} - \frac{\dot{q}D}{4h}$$

$$\text{and } T_s = 600\,[K] - \frac{10^6 \times 0.01}{4\times 25}\,[K] = \underline{500\,[K]}$$

(iii): $q'' = h(T_m - T_s) = h(T_{mi} - T_s) = 25(600-500) = 2500\left[\frac{W}{m^2}\right]$

(or: $q''\pi D = \dot{q}\frac{\pi D^2}{4} \Rightarrow q'' = \frac{\dot{q}D}{4} = \frac{10^6\times 0.01}{4} = 2500\left[\frac{W}{m^2}\right]$

(iv): $q'' = \epsilon\sigma(T_s^4 - T_\infty^4)$

but $T_\infty = 0$. so $\Rightarrow \epsilon = \frac{q''}{\sigma T_s^4} = \underline{0.71}$

(Note: The outer and inner surfaces of the tube are assumed to have ~~the~~ same temperature)