EE117A SOLUTION -- *Fall1997*

Electromagnetic Field and Waves

Wed, Oct 29, 1997
Costas J. Spanos

NOTE: This solution is from a student who got 30/30, 30/35, and 35/35, that is total 95 out of 100.

PROBLEM 1 OF 3 (30 Points)

State (in words) the Gauss theorem (7pts)

The flux of electric field, if integrated over a surface that enclosed a volume, equals to the total of charge enclosed by that volume divided by the dielectric(*E*) of material which **E** (of interest) is in.

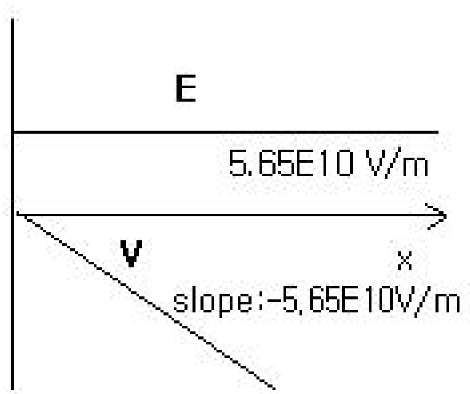
What is the differential form of Ampere theorem? (7pts)

the cross product of gradient vector and H vector equals to J vector, that is
gradient(Vector) X H(Vector) = J(Vector)

What is an "irrotationl" field? (6pts)

The Field F(Vector) is irrotational when the cross product of gradient(Vector) and F(Vector) equals to zero, that is gradient(Vector) X F(Vector) = 0. (This case is usually true for static **E** Field.)

An infinite surface on the y-z plain has a charge density of 1 Cb/m^2.
Plot the intensity of the E field and the potential (referenced to infinity) as a function of x, when x > 0 and $E_o = 8.854 \times 10^{-12}$ F/m (10pts)

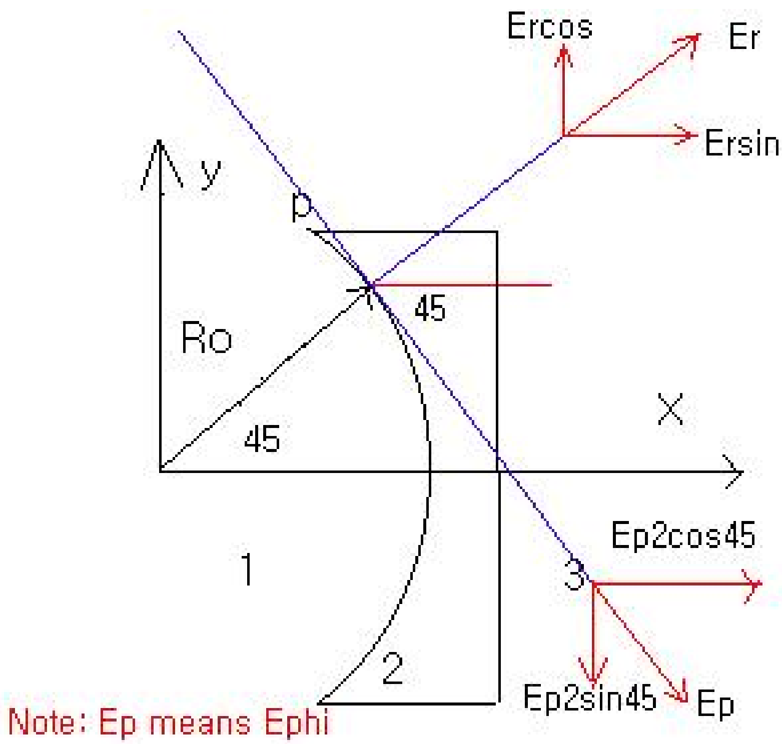


E = Sigma/(2*Epsilon$_o$) = 1 / (@*8.854E-10) = 5.65E10

Note: the value of slope in the graph can be wrong.

PROBLEM 2 OF 3 (35 points)

a) Dielectric lenses can be used to collimate electromagnetic fields. In the figure below, the left surface of the dielectric lens if that of a circular cylinder, and the right surface is a plane. If E_1 at point **P**(r_o, 45^o, z) in region 1 is e_r5-e_{phi}, what must be the dielectric constant of the lens in order the E_1 order the E_3 in region 3 is parallel to the x-axis? (15 points)



E at point p is $l_r5 - l_{phi}3$

At the bounding, E t_1 = E t_2 and E_1 E_{n1} = E_2 E_{n2}.

Therefore, in material 2, E_{phi2} = -3 l_{phi} and E_o E_{r1} = E_2 E_{r2}

At bounding 2 and 3, E t_2 = E t_3, E_2E_{n2} = E_3E_{n3}

but we want E_3 in region 3 to be parallel in x axis.

Therefore, E t_2 = E t_3 = 0

E_r cos45 = E_{phi} sin45

E_2(E_r sin45) + E_2 (E_{phi} cos45) = E_3 E_3 = E_o E_3

E_2(E_r/sqrt(2)+ E_{phi}/sqrt(2)) = E_o E_3

Note: This problem scored 10/15.
Note2: *E* means Epsilon.

b) How much energy do you need to bring 1Cb of point charge from infinity to a point 0.5 meters away from an infinite conducting plane in vacuum? (20 points)

This problem is equivalent to the problem of having an opposite charge located at the mirror position. Find the energy of this assembling form.

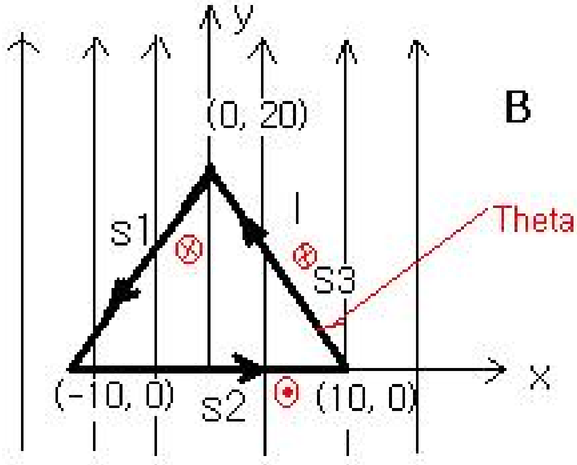
W = .5 * sumation of(Sigma) QV

= .5(Q_1V_1 + Q_2V_2)

= .5((-1*1)/(4*pi*E_o) + (-1*1)/(4*pi*E_o))

= (-1*1)/(4*pi*E_o) = (-1*1)/(4*pi*8.854E-12) = -8.99x10^9J

PROBLEM 3 OF 3 (35 points) A d-c current I = 10A flows in a triangular loop in the xy-plane as is shown below. Assuming a uniform magnetic flux density **B** = e_y6(mT) in the region, find the forces and torque on the loop. (Mu = 4* Pi* 10^{-7} H/m)



Force on Segment S1:
F = I*dl(Vector) X B(Vector) = ILB sin(theta) = 10 * 10 sqrt(5) * 6E-3 * (1/sqrt(5)) = 0.6 N
Direction: comes out of the paper

Force on Segment S2:
F = I*dl(Vector) X B(Vector) = ILB sin(theta) = 10 * 20 * 6E-3 = 1.2 N
Direction: go through the paper

Force on Segment S3:
F = I*dl(Vector) X B(Vector) = ILB sin(theta) = 10 * 10 sqrt(5) * 6E-3 * (1/sqrt(5)) = 0.6 N
Direction: comes out of the paper

Total Force and Total Torque:
Total force = 1.2 - 0.6 - 0.6 = 0 N
Total torque = F(Vector) X L(Vector) = 1.2 x 20 = 24 Nm