Professor Vetterli

EECS123 - Midterm 2

15 November 1994

8:10 - 9:30 a.m.

This is an open book exam. Calculators are allowed. Please show your work clearly if you wish to receive partial credit. Good Luck!

(30 pts)

1. Consider the following discrete-time system

a) Give the transfer function H(z) of this system, and sketch the magnitude response | H(e^jw) | . What is the phase behavior (linear, minimum, maximum, or arbitrary phase)?

(6 pts)

b. Find a casual and stable inverse filter G(z) such that H(z) G(z) has a flat magnitude response, or

| H(e^jw) G(e^jw) | = 1

(6 pts)

c. Is G(z) above unique ? If not, give another solution.

(6 pts)

d. Find a causal and stable filter F(z) such that H(z) F(z) has linear phase (that is, its impulse response is either symmetric or antisymmetric). If there is more than one solution, pick the one that minimizes the degree of H(z) F(z).

(6 pts)

e. Same as (d) above, but restrict F(z) to be an FIR filter. Give a minimum degree solution, as well as a non-minimum degree one.

(35 pts)

2. Consider zero phase type I filters with real coefficients, that is, filters that are symmetric round the origin and have N=2L + 1 real valued taps.

(10 pts)

a. Assume the filter has a z-transform H(z) = z +3/2 + z^-1. Show that this filter is an optimum minimax design for a lowpass filter with desired response

$Hd(e^{jw}) = 3; 0 \leq w \leq w(p)$

$0; w(s) \leq w \leq \pi$

and maximum error $\delta = ½$

In particular, indicate:

i. What are w(p) and w(s)?
ii. What are the alternation frequencies?
iii. Is this an "extraripple" solution? (Hint: Plot $H(e^{jw})$ for $0 \leq w \leq \pi$.)

(10 pts)

b. From H(z), devise a new filter $G(z) = H(z) + \delta$

i. Show that $G(e^{jw}) \geq 0$, and for what wo, $G(e^{jwo}) = 0$.
ii. Because $G(e^{jw}) \geq 0$, G(z) can be factored as

$G(z) = R(z) R(z^{-1})$.

Given R(z) in this case.

(10 pts)

c. Assume a half band lowpass filter F(z) designed with the Parks-Ms-Clellan algorithm. Assume further that N=9 and that it has half the alternation in the pass band ($0….0.4\pi$) and the other hapf in the stop band ($0.6\pi …….\pi$). From this filter, derive $Fp(z) = F(z) + \delta$, where δ is mazimum error in the stopband. Sketch $Fp(e^{jw})$ for $0 \leq w \leq \pi$.

(5 pts)

d. In part (b), we stated that if G(z) is such that $G(e^{jw}) \geq 0$, then $G(z) = R(z) R(z^{-1})$. Show the converse, namely that if $G(z) = R(z) R(z^{-1})$, then necessarily $G(e^{jw}) \geq 0$.

(35 pts)

3. Consider a continuous time filter with impulse response

$hc(t) = 1; |t| \leq \tau$

0; otherwise

The Fourier Transform of hc(t) is a sinc function given by the formula

$Hc(j\Omega) = 2\sin\Omega \tau / \Omega$

For parts a-d below, consider applying the bilinear transformation to $Hc(j\Omega)$ to derive a discrete time filter $Hb(e^{jw})$. Use $Td = \tau / M$.

(5 pts)

a. Find an expression for he zeros of Hb(e^jw). Sketch H(b) (e^jw).

(5 pts)

b. At what frequency w(p) does |Hb(e^jw)| evluated at (w = w(P)) =

$$2/\pi * \max | Hb (e^{jw})| ?$$

(5 pts)

c. Find the width (in radians) of the main lobe of Hb(e^jw). The main lobe is the lobe centered around w=0 and the width refers to the distance betwen the zeris flanking the main lobe.

(5 pts)

d. Using the characteristics of Hb(e^jw) and the properties of the DTFT, determine if the discrete time impulse response hb [n] is (1) FIR; (2) symmetric; (3) stable; (4) causal; (5) real. (explain your answers.)

For parts e -f below, consider applying impulse invariance to Hc(jΩ) to derive a discrete time filter Hi(e^jw). Again use Td = τ / M. Suggestion: Do this in the frequency domain.

(8 pts)

e. Are there any frequencies at which aliasing does not affect the resultant filter Hi(e^jw)? If so, find them. In other words, determine any values of w for which

Hi(e^jw) = Hc(j w/Td).

(7 pts)

f) Sketch Hi(e^jw) and find the width of the main lobe.

EE123 Solution

Problem1

a. H(z) = 1 +7/2 Z^-1 + 3/2 Z^-2

H(z) = (1 + 3Z^-1) (1+1/2 Z^-1) (fig 1)

first term : zero @ Z=-3 outside U.C. (therefore not min phase)

2^{nd} term : zero @ z=-1/2 inside U.C. (therefore not maz phase)

h[n] is not symmetric, therefore not linear phase

è arbitrary phase (fig 2)

b. |H(e^jw) G (e^jw)| = 1

This product is an allpass, so its poles and zeros must be in reciprocal conjugate pairs.

G(z)

(fig 3)

The pole @ z = -1/2 cancels the zero of H(z) @ z=-1/2.

The pole @ z = -1/3 combines with the zero @ z =-3 to make an allpass filter.

G(z) = 1/3 * 1/[(1+1/2 Z^-1) (1+1/3 Z^-1)]

This is the important part; it takes care of the magnitude equalization. The scale factor of 1/3 normalizes the magnitude to 1.

c. G(z) is not unique. Consider

G2(z) = G(z) Hap(z)

G(z) from above

Hap(z) allpass filter

Then |H(e^jw)G2(e^jw)| = |H(e^jw)G(e^jw)|*| |Hap(e^jw)| = 1

(fig 4)

We can canstruct G2(z) by adding reciprocal conjugate pole-zero pairs to G(z). Such pole-zero pairs constitute an allpass filter.

G2(z) = 1/3 * ½ * (1-2 Z^-1) / [(1+ 1/2 Z^-1) (1+ 1/3 Z^-1) (1-1/2 Z^-1)]

d. linear phase = è symmetry or anit-sym è zeros are reciprocal paris

H(z) (fig 5)

F(z) (fig 6) causal, stable all poles and zeros are inside u.c.

H(z) F(z) (fig 7)

F(z) = (1+ 1/3 Z^-1) / (1+ 1/2 Z^-1)

e. If F(z) is restricted to be FIR, it can't have any poles except @ z=0 or z = infinite.We need to construct F(z) so that H(z) F (z) has zeros in reciprocal pairs

F(z) (fig 8)

H(z) F(z) (fig 9)

F(z) = (z+2)(z+ 1/3)

F(z) = Z^2 + 7/2 z + 2/3 minimum degree solution

F2(z) (fig 10) or F3(z) , etc (fig 11)

F2(z) = (Z^2 + 7/2 z + 2/3) (z+1)

F2(z) = Z^3 + 10/3 Z^2 + 9/3 z + 2/3

Problem 2

Real zero-phase type I filter N = 2L +1

a. H(z) = z + 3/2 + z^-1

H(e^jw) = e^jw + e^-jw + 3/2

H(e^jw) = 2cosw + 3/2

(fig 12)

i. wp = π /3 ws = 2π /3
ii. {0, π /3, 2π /3, π }
iii. yes, this is an extraripple soultion. It has alternations at 0 and π .

b. G(z) = H(z) + δ = z + 2 + z^-1

i. G(e^jw) = 2cosw +2

= 2(cosw + 1) >= 0 for all w

G(e^jwo) = 0 for wo = π

ii. G(z) = R(z) R(z^-1)

= z + 2 + z^-1

= (1 + z) (1 + z^-1)

R(z) = 1+ z (or R(z) = 1 + z^-1)

c. N = 9 = è L = 4

half of alternations n passband | implies an even number of alternations r = L + 2 = 6

half of alternations in stopband |

Adding the stopband error δ shifts the response up so that Fp(e^jw) >= 0 for all w.

(fig 13) alternations of F(e^jw) are makred with x's

d. G(z) = R(z) R(z^-1)

G(e^jw) = R(e^jw) R(e^-jw)

R(e^jw) is conjugate symemetric since r[n] is real, so R(e^-jw) = R*(e^jw)

G(e^jw) = R(e^jw) R(e^jw)

G(e^jw) = |R(e^jw)|^2 >= 0 for all w.

problem 3

a. Hc(jΩ) = 2 sinΩ π /Ω

The zeros of Hc(jΩ) are at Ω π = π k { k not equal to 0}

Ω = π k / τ .

To find the zeros of Hb(e^jw), find where the zeros of Hc(jΩ) are mapped to using the bilinear transformation equation (7.28b)

w = 2 arctan (Ω Td/2) Td = τ / M

= 2 arctan (π k / 2M) {k not equ to 0}

b. Since the bilinear transformation is a one to one mapping, we can solve this problem in Ω and map it to w.

|Hc(jΩ p)| = 2/π maz |Hc(jΩ)| = 2τ

2 |sin Ω pτ | / Ω p = 4τ /π

By inspection

Ω pπ = π /2 , Ω p = π /2τ

wp = 2 arctan(Ω pTd/2)

= 2 arctan (π /2τ * ½ * τ /M)

= 2 arctan (π /4M)

c. The first zero of Hc(jΩ) is at Ω = π /τ

The first zero of Hb(e^jw) is at w = 2 arctan (π /2M)

So the width of the main lobe is 4 arctan(π /2M)

d. (1) Not FIR H3(e^jw) has an infinite number of zeros

2. Symmetric Hb(e^jw) is real
1. Stable No poles on the U.C.
2. Not casual Since it's symmetric and not FIR, it mush be not causal
3. Real Hb(e^jw) is conjugate symmetric

I left out the sjetch for part (a)…..

(fig 14)

Since the entire jΩ axis is mapped into the w range [-π ,π], the zeros get closer and closer together as |w| à π . (there are an infinite number of zeros.)

e. For the impulse invariance method, we have

Hi(e^jw) = ∑ (k = - infinite to + infinite) Hc(j w/Td + j 2π k/Td)

= ∑ (k) sin [(w/Td + 2π k/Td) τ] [2/(w/Td + 2π k/Td)] ;Td=τ /M

= ∑ (k) sin [(wM+2π kM)*τ /τ] [(2τ /M)/(w+2π k)]

= ∑ (k) sin (wM+2π kM) [(2τ /M)/(w+2π k)] ; 2π kM – integer multiple

of

2π

= sin(wM) ∑ (k) [(2τ /M)/(w+2π k)] ; sin(wM) – zeros @ w=π n/M,

except n=0

|Hi(e^jw)| w=0 = lim(w à 0) sin(wM) ∑ (k) [(2τ /M)/(w+2π k)] = 2τ

We know that Hc(jΩ) has zeros @ Ω =π n/τ .

Note Ω Td = π n/τ * τ /M = π n/M = zeros in w

So Hi(e^jw) = Hc(jw/Td) at w=π n/M

f. (fig 15)

Main lobe width = 2π /M