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8:10-9:00, Friday, March 23, 2007

ME 106 FLUID MECHANICS

EXAM 2 – open book, open notes, no external communication

1.(15+15=30%)

Consider the flow past a sphere at very low speeds, where the inertial forces can be neglected compared to the viscous forces. Hence we can judiciously ignore the dependence of the total drag on the density of the fluid. Ignore gravity.

(a) List the relevant flow variables in determining the drag of the sphere.

(b) Form the relevant dimensionless parameter(s).

Analytic solution of the problem.

$$D = 3\pi\mu dU$$

$$D = fn(\mu, U, d) \implies D \sim \mu^\beta U^\gamma d^\delta \implies MLT^{-2} = [ML^{-1}T^{-1}]^\beta \times [LT^{-1}]^\gamma \times L^\delta$$

$$\begin{array}{lrcl} M: & 1 = \beta & & \beta = 1 & & \\ L: & 1 = -\beta + \gamma + \delta & \implies & \gamma = 1 & \implies & D \sim \mu d U \quad D/\mu dU = constant = 3\pi \\ T: & -2 = -\beta - \gamma & & \delta = 1 & & \end{array}$$

2.(10+10+10+5=35%)

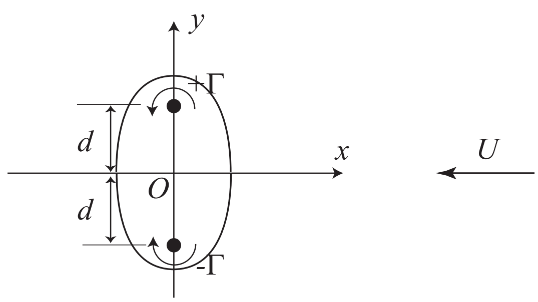
Consider the potential flow field of a point vortex of strength Γ at a distance d over a flat plate in an otherwise uniform flow of velocity $(-U, 0)$. This flow field can be modeled as two counter-rotating vortices $(+\Gamma,\ -\Gamma)$ placed symmetrically across the plane as shown in the figure. Due to flow symmetry, the plane $(y = 0)$ becomes a streamline. For the purpose of the calculations here, let us take $U = \Gamma/4\pi d$.

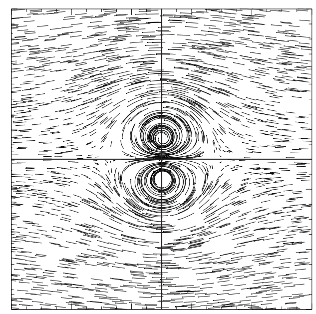
(a) Using superposition, determine the velocity at the origin, $(x, y) = (0, 0)$.

(b) Construct the potential function or the stream function of the combined flow field.

(c) Determine the stagnation points (you may use either superposition, or your result above).

(d) Sketch the closed streamline passing through the stagnation points (you don't have to calculate its actual shape).





Summing velocity vectors at the origin

$$\mathbf{u}_o = (u, 0) = \left(\frac{\Gamma}{2\pi d} + \frac{\Gamma}{2\pi d} - U,\ 0\right) = \left(\frac{\Gamma}{2\pi d} + \frac{\Gamma}{2\pi d} - \frac{\Gamma}{4\pi d},\ 0\right) = \left(\frac{3\Gamma}{4\pi d},\ 0\right) \tag{1}$$

In an (x, y) coordinate system where the vortices are at $(x, y)_{\pm\Gamma} = (0, \pm d)$ stagnation points occur along the x-axis where the induced velocity due to both vortices is balance by the uniform flow.

$$U = 2 \times \frac{\Gamma}{2\pi\sqrt{d^2 + x_s^2}} \times \frac{d}{\sqrt{d^2 + x_s^2}} \Longrightarrow (x, y)_s = (\pm\sqrt{3}d, 0) \tag{2}$$

Total potential function construction

$$\phi = \phi_U + \phi_{+\Gamma} + \phi_{-\Gamma} = -Ux + \frac{\Gamma}{2\pi}\, tan^{-1}\left(\frac{y - d}{x}\right) - \frac{-\Gamma}{2\pi}\, tan^{-1}\left(\frac{y + d}{x}\right) \tag{3}$$

Total stream function construction

$$\psi = \psi_U + \psi_{+\Gamma} + \psi_{-\Gamma} = -Uy - \frac{\Gamma}{2\pi}\, ln\sqrt{x^2 + (y - d)^2} - \frac{-\Gamma}{2\pi}\, ln\sqrt{x^2 + (y + d)^2} \tag{4}$$

Closed streamline through the stagnation points

$$\psi = \frac{\Gamma}{4\pi}\, ln\left[e^{-y/d}\frac{x^2 + (y + d)^2}{x^2 + (y - d)^2}\right] = 0 \tag{5}$$

The closed streamline is an oval with x-axis intercepts at $\pm 1.55d$

3.(5+5+5+10+10+5=40%)

Consider a piston of diameter $D = 2a$ and length L moving at a constant speed U in to a cylinder. The cylinder contains a fluid of density ρ and viscosity μ. The gap between the cylinder and the piston is d such that $d \ll D$. Therefore, the flow in the gap can be treated as that between two parallel plates. Flow is incompressible. Assume the flow is fully developed along the length of the piston. Ignore the end effects and gravity.

(a) Determine the *leakage* flow rate.

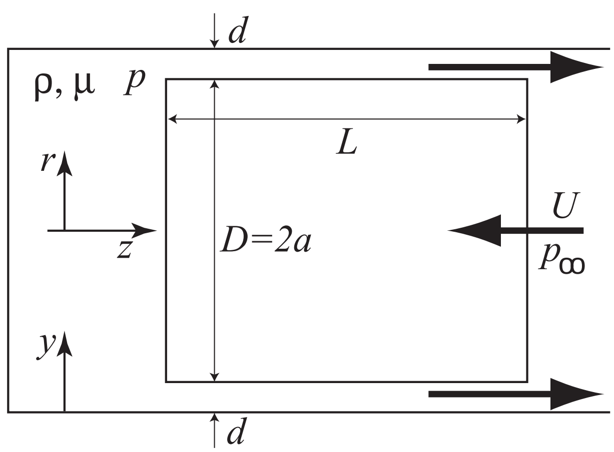
(b) State the velocity boundary conditions on the piston and cylinder.

(c) Roughly sketch the velocity profile in the gap between the cylinder and the piston.

(d) Write down the equations of motion describing the flow in the gap.

(e) Solve for the velocity profile, assuming a constant pressure gradient along the piston.

(f) Finally, determine the pressure rise $p - p_\infty$ in the cylinder. *Hint: Use the flow rate resuts from (a) and (e).*



leakage flow rate $Q = \pi a^2 U$

define

$$P = \frac{1}{\mu}\frac{dp}{dz}$$

Thin gap approximation: Plane Couette Flow, u $= [u(y), 0, 0]$

$$\frac{d^2u}{dy^2} = P \Longrightarrow u(y) = \frac{1}{2}P\,y^2 + Ay + B$$

$$u(0) = 0,\ u(d) = -U \Longrightarrow u(y) = \frac{d^2P}{2}\left(\frac{y^2}{d^2} - \frac{y}{d}\right) - U\frac{y}{d}$$

$$Q = 2\pi a \times \int_o^d u(y)dy = -2\pi a\left(\frac{d^3P}{12} + \frac{Ud}{2}\right) = \pi a^2 U \Longrightarrow P = -\frac{6(a-d)U}{d^3}$$

$$u(y) = -\frac{3(a-d)U}{d}\left(\frac{y^2}{d^2} - \frac{y}{d}\right) - U\frac{y}{d}$$

$$p - p_\infty = -L \times \frac{dp}{dz} = -\mu LP = +\frac{6\mu(a-d)LU}{d^3}$$