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Departments of Mechanical Engineering and Materials Science & Engineering

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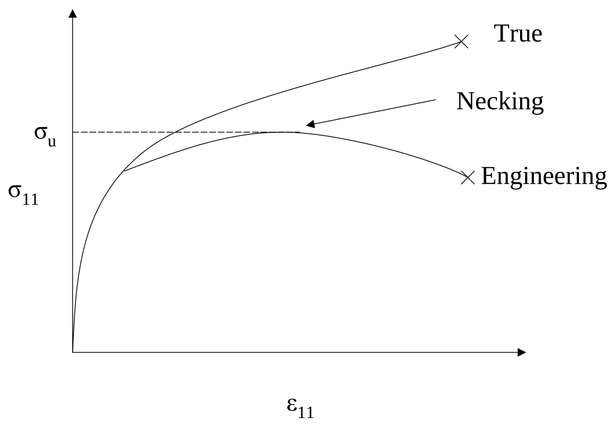
C-ME 124/C-MSE 113
Mechanical Behavior of Materials
Midterm Exam #2 Solutions

Problem 1:

You are given a piece of ductile polycrystalline metal, such as virgin copper, with uniform cross-sectional area that behaves according to the following constitutive behaviour:

$$\sigma_{11} = k\epsilon_{11}^{n}$$

where σ_{11} and ϵ_{11} are respectively the uniaxial(true) stress and plastic strain.
a)If this material is pulled in uniaxial tension, draw and label the following on a Stress-Strain graph:(10 points)
i)variation in engineering stress vs.engineering strain
ii)the corresponding variation in true stress vs.true strain
iii)indicate where necking occurs
iv)label the ultimate tensile strength,σ_u



b)Briefly explain(3 sentences or less) the difference between the true stress-strain and the engineering stress-strain diagrams.(5 points)

True Stress-Strain compensates for changing cross-sectional area,while Engineering Stress-Strain uses the Original Area.

c)By stating your assumptions clearly,derive the following relationships:
i)An expression relating true stress to engineering stress and engineering strain only(5 points)
Assume Constant Volume:
$A_0 l_0 = Al$
$\frac{A_0}{A} = \frac{l}{l_0}$
$\epsilon_{eng} = \frac{l-l_0}{l_0} = \frac{l}{l_0} - 1$
$\frac{l}{l_0} = 1 + \epsilon_{eng}$

Also, $\sigma_{eng} = \frac{P}{A_0}$, $\sigma_T = \frac{P}{A}$

$\sigma_T = \sigma_{eng}\frac{A_0}{A} = \sigma_{eng}\frac{l}{l_0}$
$\sigma_T = \sigma_{eng}(1 + \epsilon_{eng})$

ii)an expression relating true strain to engineering strain only(5 points)
$d\epsilon_T = \frac{dl}{l}$
$\epsilon_T = \int_{l_0}^{l} (\frac{1}{l})\, dl = ln(\frac{l}{l_0})$
$\epsilon_T = ln(1 + \epsilon_{eng})$
d)Briefly describe(3 sentences or less) what occurs microstructurally at the onset of:(5 points)
i)Plastic Deformation
Dislocation mobility and bond breaking
ii)Necking
Geometric Softening

e)Based on the constitutive law stated above,derive the true and engineering strains at the onset of necking.Again, state your assumptions clearly.(10 points)

At Necking:
dP=0
$P = \sigma_T A$
$dP = 0 = Ad\sigma_T + \sigma_T dA$
$\frac{\sigma_T}{d\sigma_T} = -\frac{A}{dA}$

Also assuming Constant Volume:
$dV = 0$
$V = lA$
$dV = 0 = Adl + ldA$
$-\frac{A}{dA} = \frac{l}{dl}$

So,
$\frac{\sigma_T}{d\sigma_T} = \frac{l}{dl}$

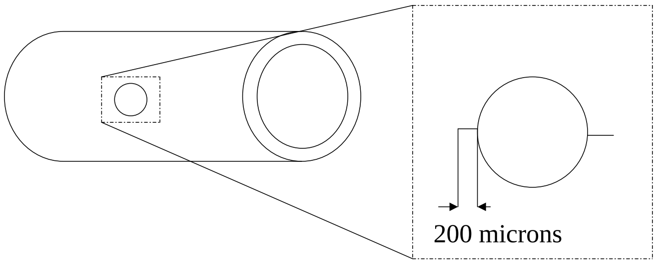
Also since $d\epsilon_T = \frac{dl}{l}$,
$\frac{\sigma_T}{d\sigma_T} = \frac{1}{d\epsilon_T}$
$\sigma_T = \frac{d\sigma_T}{d\epsilon_T}$
$k\epsilon_{11}^n = \frac{d(k\epsilon_{11}^n)}{d\epsilon_{11}}$
$k\epsilon_{11}^n = nk\epsilon_{11}^{n-1}$

$\epsilon_T = n$. Also, $\epsilon_T = ln(1 + \epsilon_{eng})$

$\epsilon_{eng} = exp(n) - 1$

Problem 2

A pressure tube is being put in service and the users wish to monitor the gas flux through the tube using a flow-meter.A 1 mm diameter hole is therefore drilled into the side of the tube to accommodate the flow-meter inlet.When removing the drill bit, the machinist accidentally nicked the edges of the hole to form microscopic cracks of length, 200 μm, oriented in the longitudinal direction of the tube.(see diagram below).A high strength ferritic steel is used to make the tube, which has the following mechanical properties:yield stress σ_y= 830 MPa, modulus of elasticity E=200 GPa, Poisson's ratio ν=0.32, and a fracture toughness K_{Ic} of 30 MPa$\sqrt{m}$.The tube dimensions are 15 m long, 12 cm outer diameter, and 2.5 mm wall thickness.Pressure in the tube is at 30 MPa (you may assume that the pressure source can be maintained constant within the tube despite the possible pressure release through the cracks).By considering the stress concentration around the hole,calculate the stress intensity factor developed at the machining cracks.Will the tube fail upon pressurization?Is your answer conservative?Support your answer mathematically.



Solution

$\frac{r}{t} = \frac{6}{0.25} = 24 >> 1$

Therefore the thin walled approximation applies

$\sigma_{rr} = \sigma_{zz} = \sigma_{\theta z} = \sigma_{\theta r} = \sigma_{rz} = 0$

$\sigma_{\theta\theta} = \frac{pr}{t} = 30MPa(24) = 720MPa(7 \text{ points})$

Treating this as a hole in an infinite plate under tension,

$K_I = \sigma\sqrt{\pi l}F(\frac{l}{R})$

Where, $\frac{l}{R} = \frac{200x10^{-6}}{0.5x10^{-3}} = 0.4$

$F(\frac{l}{R}) = 1.96$

$K_I = 1.96\sigma_{\theta\theta}\sqrt{\pi l} = 1.96(720MPa)\sqrt{\pi(200x10^{-6})}$

$K_I = 35MPa\sqrt{m}(7 \text{ points})$

Will failure occur?

1)Yield Criteria:(3 points)

Because there is only one principal stress,Tresca and von Mises reduce to the same criterion:

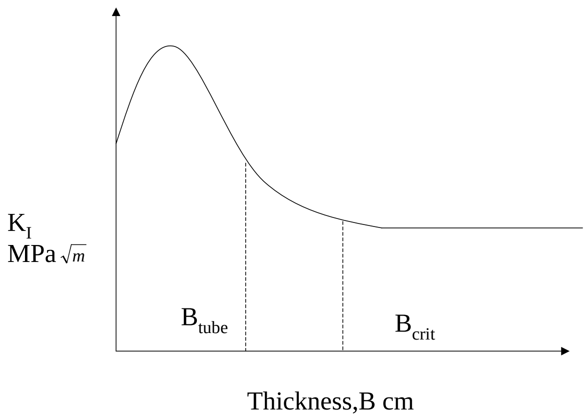
if $\sigma_{\theta\theta} < \sigma_y$, then yielding will not occur.

Check: $720MPa <$ 830 MPa, thus no yielding.

2)Fracture Criteria:(3 points)
If $K_I < K_{Ic}$ then fracture will not occur.
Check:$35MPa\sqrt{m} > 30MPa\sqrt{m}$.
Thus, Fracture will occur!

Is this conservative?

We need ss plot.(5 points)



$r_y < \frac{1}{15}B_{crit}$, where B is the wall thickness and r_y is the plastic zone size.

$\frac{1}{2\pi}(\frac{K_{Ic}}{\sigma_y})^2 < \frac{1}{15}B_{crit}$

$B_{crit} > \frac{15}{2\pi}(\frac{K_{Ic}}{\sigma_y})^2 = \frac{15}{2\pi}(\frac{30MPa\sqrt{m}}{830MPa})^2$

$B_{crit} > 0.31cm$
Therefore we underestimated the toughness, thus our conclusions are conservative.(5 points)

Problem 3

A nuclear pressure tube, made of annealed Zircaloy 2 alloy, is pressurized with helium gas at 300 psi for 60 days at 700°F.It is then subjected to an increase in internal pressure to 600 psi for 30 days at 650°F.If the tube has an initial wall thickness of 1/8 inches and an outer diameter of 3 inches,calculate the final dimensions(diameter and wall thickness) after this 90 day period.You may neglect elastic and primary creep strains and assume a constitutive law for Zircaloy 2 between 600 and 800°F to be:

$$\dot{\epsilon} = [(\sigma/\sigma_0)^{16}]exp[(-Q)/kT]$$

where σ_0=5.5 ksi and Q = 1.065x10^{-18} in.lb., k = 6.79x10^{-23} in.lb/°R is Boltzmann's Constant and T is absolute temperature(in °Rankine) °R=°F+460.

Hint:use $\epsilon_{\theta\theta} = ln(r/r_o) = \dot{\epsilon}_{\theta\theta}t$

Solution

$\frac{r}{t} = \frac{1.5}{0.125} = 12 >> 1$

Therefore the thin walled approximation applies

Stresses:(5 points)

$\sigma_{rr} = \sigma_{zz} = \sigma_{\theta z} = \sigma_{\theta r} = \sigma_{rz} = 0$

$\sigma_{\theta\theta} = \frac{pr}{t} = 12p$

Strains:(5 points)

$\epsilon_{zz} = 0,$

$\epsilon_{rr} = ln(\frac{t}{t_0}) = 0$ as $\sigma_{rr} = 0$

$\epsilon_{\theta\theta} = ln(\frac{r}{r_0})$

Strain Rate:

$\dot{\epsilon}_{\theta\theta} = [(\frac{\sigma_{\theta\theta}}{\sigma_0})^{16}]exp[\frac{-Q}{kT}]$
$\dot{\epsilon}_{\theta\theta} = [(\frac{\sigma_{\theta\theta}}{5.5})^{16}]exp[\frac{-1.065x10^{-18}}{6.79x10^{-23}T}]$
$\dot{\epsilon}_{\theta\theta} = [(\frac{\sigma_{\theta\theta}}{5.5})^{16}]exp[\frac{-1.568x10^{4}}{T}]$

For the first temperature excursion:
p=300 psi so $\sigma_{\theta\theta} = 3.6$ ksi
$T = 700$°F=1160°R
t=60 days=1440 hrs
Since Creep Rate is constant for steady state creep:
$\epsilon_{\theta\theta} = \dot{\epsilon}_{\theta\theta}$x t
$\epsilon_{\theta\theta} = (\frac{3.6}{5.5})^{16}exp(\frac{-1.568x10^{4}}{1160})$x1440=2.2 x10^{-6}(7 points)
For the second temperature excursion:
p=600 psi so $\sigma_{\theta\theta} = 7.2$ ksi
$T = 650$°F=1110°R
t=30 days=720 hrs

Since Creep Rate is constant for steady state creep:
$\epsilon_{\theta\theta} = \dot{\epsilon}_{\theta\theta}$x t
$\epsilon_{\theta\theta} = (\frac{7.2}{5.5})^{16}exp(\frac{-1.568x10^{4}}{1110})$x720=3.9 x10^{-2}(7 points)

since,
$\epsilon_{\theta\theta} = ln(\frac{r}{r_0}) = ln(\frac{r}{1.5})$=3.9 x 10^{-2}

The final outer radius is 1.56"(3 points)
The final wall thickness is the same as the initial wall thickness(0.125") as $\epsilon_{rr} = 0$(3 points)