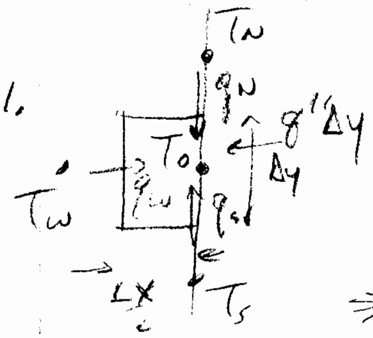
Midterm Fall 2001 Solutions

1.



$$\frac{dE_{cv}}{dt} = \dot{E}_{in} - \cancel{\dot{E}_{out}}^{0} + \cancel{\dot{E}_g}^{0}$$

$$\rho c V \frac{\partial T}{\partial t} = q_W + q_N + q_S + q''\Delta y$$

$$\Rightarrow \frac{1}{2}\rho c \Delta x \Delta y \left(\frac{T_0^{p+1} - T_0^p}{\Delta t}\right) = k\Delta y\left(\frac{T_W^p - T_0^p}{\Delta x}\right)$$

$$+ k\left(\frac{\Delta x}{2}\right)\left(\frac{T_N^p - T_0^p}{\Delta y}\right) - k\left(\frac{\Delta x}{2}\right)\left(\frac{T_S^p - T_0^p}{\Delta y}\right) + q''\Delta y$$

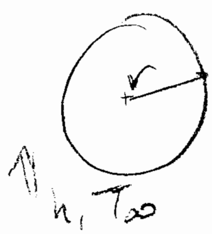
$$\Rightarrow T_0^{p+1} - T_0^p = \frac{2k\left(\frac{\Delta y}{\Delta x}\right)\Delta t}{\rho c \Delta x \Delta y}\left(T_W^p - T_0^p\right) + \frac{k\left(\frac{\Delta x}{\Delta y}\right)\Delta t}{\rho c \Delta x \Delta y}\left(T_N^p + T_S^p - 2T_0^p\right)$$

$$+ \frac{q''\Delta y \Delta t}{\rho c \Delta x \Delta y}$$

$$\boxed{\Rightarrow T_0^{p+1} = \frac{2\alpha\Delta t}{\Delta x^2}T_W^p + \frac{\alpha\Delta t}{\Delta y^2}\left(T_N^p + T_S^p\right) + \frac{q''\Delta t}{\Delta x} + \left(1 - \frac{2\alpha\Delta t}{\Delta x^2} - \frac{2\alpha\Delta t}{\Delta y^2}\right)T_0^p}$$

STABILITY CRITERION: $1 - 2\left(\frac{\alpha\Delta t}{\Delta x^2} + \frac{\alpha\Delta t}{\Delta y^2}\right) > 0$

2.



$r = 1\ mm = 0.001\ m$

$h = 100\ \frac{W}{m^2\cdot K}$, $T_\infty = 1000\ K$, $T_i = 300\ K$

$k = 10\ \frac{W}{m\cdot K}$, $\rho c = 3\times10^6\ \frac{J}{m^3\cdot K}$

What is t for $T_{FINAL} = 900\ K$?

$$Bi = \frac{hL_c}{k}, \quad L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = r/3 = 0.0003$$

$$\Rightarrow Bi = \frac{h(r/3)}{k} = 0.003 < 0.1 \quad \therefore \text{ USE L.C.}$$

$$\frac{dE_{cv}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_g}^{\,0}$$

$\frac{dE_{cv}}{dt} \rightarrow q_{conv}$, CV

$$\rho c V \frac{dT}{dt} = -hA_s(T - T_\infty)$$

LET $\theta \equiv T - T_\infty$, $d\theta = dT$

$$\Rightarrow \frac{d\theta}{dt} = \frac{-hA_s\theta}{\rho Vc} \rightarrow \frac{d\theta}{\theta} = \frac{-hA_s}{\rho Vc}\,dt$$

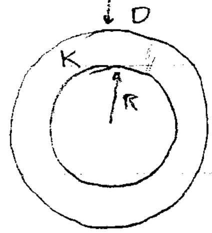
$$\Rightarrow \ln\theta\Big|_{\theta_i}^{\theta} = \frac{-hA_s}{\rho Vc}\Big|_0^t \rightarrow \ln\left(\frac{\theta}{\theta_i}\right) = \frac{-hA_s t}{\rho Vc}$$

$$\rightarrow t = \frac{\rho Vc}{hA_s}\ln\left(\frac{\theta_i}{\theta}\right) = \frac{\rho Vc}{hA_s}\ln\left(\frac{T_i - T_\infty}{T_{FINAL} - T_\infty}\right)$$

$$\boxed{t = 19.46\ \text{sec}}$$

p 109 use R_{OUT}

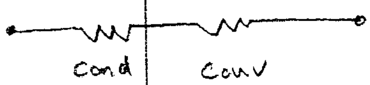
③ Thin Tube



Change D, minimize Resistance

① Derive an Expression for Resistance

Tube Conduction $\dfrac{\ln\left(\frac{r_{out}}{r_{in}}\right)}{2\pi L k}$



$r_{in} = R$

$r_{out} = R + D$

$1 + \frac{D}{R}$

Tube Convection : $\dfrac{1}{2\pi L r_{out} h}$

$$R_{TH} = \frac{\ln\left(\frac{R+D}{R}\right)}{2\pi L k} + \frac{1}{2\pi L (R+D) h}$$

$$\frac{dR_{TH}}{dD} = \frac{1}{2\pi L k}\left(\frac{R}{R+D}\right)\left(\frac{1}{R}\right) - \frac{1}{2\pi L (R+D)^2 h} = 0$$

$$= \frac{1}{\cancel{2\pi} k (R+D)} - \frac{1}{\cancel{2\pi} (R+D)^2 h} = 0$$

$$R + D = \frac{k}{h}$$

$$\boxed{D_{min} = \frac{k}{h} - R}$$

Minimum?

$$\frac{d^2 R_{TH}}{dD^2} = -\frac{1}{2\pi k (R+D)^2} + \frac{1}{\pi (R+D)^3 h}$$

$$\left.\frac{d^2 R_{TH}}{dD^2}\right|_{D = \frac{k}{h} - R} = -\frac{1}{2\pi k \left(R + \frac{k}{h} - R\right)^2} + \frac{1}{\pi \left(R + \frac{k}{h} - R\right)^3 h}$$

$$= -\frac{1}{2\pi k \left(\frac{k}{h}\right)^2} + \frac{1}{\pi \left(\frac{k}{h}\right)^3 h}$$

$$= -\frac{h^2}{2\pi k^3} + \frac{h^2}{\pi k^3}$$

$$= \frac{h^2}{2\pi k^3} > 0$$ Concave Up Minimum