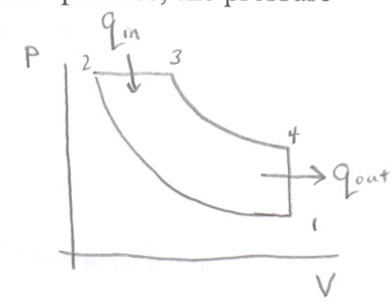
Name: ______________________
SID: ______________________

Question 1: [25 points]

An air-standard diesel cycle has a compression ratio of 20, and the heat transferred to the working fluid per cycle is 1800 kJ/kg. At the beginning of the compression process, the pressure is 0.1 MPa and the temperature is 15 degrees C. Determine



a) the pressure and temperature at each point in the cycle
b) the thermal efficiency

Cp = 1.004 kJ/kg*K
Cv = 0.717 kJ/kg*K
k = 1.4
R = 0.287 kJ/kg*K

$q_{in} = 1800 \frac{kJ}{kg}$

$P_1 = 100 \text{ kPa}$

$T_1 = 288 \text{ K}$

$r = \frac{V_1}{V_2} = 20$

a) Pressure & temperature @ each point

$$V_1 = \frac{RT_1}{P_1} = \frac{(0.287 \frac{kPa \cdot m^3}{kg \cdot K})(288 \text{ K})}{100 \text{ kPa}} = 0.82656 \frac{m^3}{kg}$$

$$P_1 V_1^k = P_2 V_2^k \rightarrow P_2 = P_1 \left(\frac{V_1}{V_2}\right)^k = (100 \text{ kPa})(20)^{1.4} \Rightarrow \boxed{P_2 = 6628.9 \text{ kPa}}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow T_2 = \frac{P_2}{P_1} \cdot \frac{V_2}{V_1} T_1 = \left(\frac{6628.9}{100}\right)\left(\frac{1}{20}\right)(228) \rightarrow \boxed{T_2 = 775.7 \text{ K}}$$

$$q_{in} = C_p(T_3 - T_2) \rightarrow T_3 = \frac{q_{in} + C_p T_2}{C_p} = \frac{(1800 \frac{kJ}{kg}) + (1.004 \frac{kJ}{kg \cdot K})(775.7 \text{ k})}{1.004 \frac{kJ}{kg \cdot K}} \Rightarrow \boxed{T_3 = 2568.5 \text{ K}}$$

$$\boxed{P_3 = P_2 = 6628.9 \text{ kPa}}$$

$$V_3 = \frac{RT_3}{P_3} = 0.1112 \frac{m^3}{kg} \qquad T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{k-1} = (2568.5 \text{ K})\left(\frac{0.1112 \, m^3/kg}{0.82656 \, m^3/kg}\right)^{0.4} \Rightarrow \boxed{T_4 = 1151.4 \text{ K}}$$

$$V_4 = V_1 \qquad \boxed{P_4 = \frac{RT_4}{V_4} = 400 \text{ kPa}}$$

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b) Thermal efficiency

$$q_{out} = C_v(T_4 - T_1) = \left(0.717 \frac{kJ}{kg \cdot K}\right)(1151.4 - 288 \text{ K}) = 619 \frac{kJ}{kg}$$

$$W_{out} = q_{in} - q_{out} = 1800 - 619 \frac{kJ}{kg} = 1180.9 \frac{kJ}{kg}$$

$$\eta_{th} = \frac{W_{out}}{q_{in}} = \frac{1180.9}{1800} \rightarrow \boxed{\eta_{th} = 0.656}$$

Name: ____________________

SID: ____________________

Question 2: [10 points]

An isolated system of total mass m is formed by mixing two equal masses of the same liquid initially at temperatures T_1 and T_2. Eventually, the system attains an equilibrium state. Each mass is incompressible with specific heat c. **Starting with the equation forms of the first and second laws of thermodynamics, show that the total entropy produced is**

$$\Delta S = mc\ln\left[\frac{T_1+T_2}{2\sqrt{T_1\cdot T_2}}\right]$$

Explain and justify each step of your derivation.

Some useful relations:

$$\ln(xy)=\ln(x)+\ln(y) \qquad \ln\left(\frac{x}{y}\right)=\ln(x)-\ln(y) \qquad \ln\left(x^y\right)=y\ln(x)$$

$$\frac{d}{dx}\left(\ln|x|\right)=\frac{1}{x} \qquad \int\frac{1}{x}dx=\ln(x)+C$$

$$\underset{T_1}{\overset{A}{\boxed{m/2}}} \quad + \quad \underset{T_2}{\overset{B}{\boxed{m/2}}}$$

1^{st} law: $\delta Q - \delta W = du$ (1)

2^{nd} law: $ds = \frac{\delta Q}{T} \rightarrow \delta Q = Tds$ (2)

Boundary work: $\delta w = Pdv$ (3)

For incompressible substance: $du = cdT$ (4)

(2), (3) and (4) into (1):

$$Tds - Pdv = cdT \longrightarrow ds = c\frac{dT}{T} + \frac{P}{T}\cancel{dv}^{=0 \text{ for incompressible}}$$

$$ds = c\frac{dT}{T}$$

Final temperature: $T_3 = \frac{1}{2}(T_1+T_2)$

For (A): $\Delta S_A = m/2\int_{T_1}^{T_3} c\frac{dT}{T} = c\frac{m}{2}\ln\left(\frac{T_3}{T_1}\right) = c\frac{m}{2}\ln\left(\frac{T_1+T_2}{2T_1}\right)$

For (B): $\Delta S_B = \frac{m}{2}\int_{T_2}^{T_3} c\frac{dT}{T} = c\frac{m}{2}\ln\left(\frac{T_3}{T_2}\right) = c\frac{m}{2}\ln\left(\frac{T_1+T_2}{2T_2}\right)$

Total entropy change:

$$\Delta S = \Delta S_A + \Delta S_B = \frac{1}{2}cm\left[\ln\left(\frac{T_1+T_2}{2T_1}\right)+\ln\left(\frac{T_1+T_2}{2T_2}\right)\right]$$

$$= \frac{1}{2}cm\left[\ln\left(\frac{(T_1+T_2)^2}{4T_1T_2}\right)\right]$$

$$= \frac{1}{2}cm\left[\ln\left(\frac{T_1+T_2}{2\sqrt{T_1T_2}}\right)^2\right]$$

$$\boxed{\Delta S = cm\left[\ln\left(\frac{T_1+T_2}{2\sqrt{T_1T_2}}\right)\right]}$$

Name: ______________________

SID: ______________________

Question 3: [25 points]

Air is compressed steadily in a reversible compressor from an inlet state of 100 kPa and 300 K to an exit pressure of 900 kPa. Determine the compressor work per unit mass for isentropic compression with k=1.4, and isothermal compression. Draw the two processes on a Pv diagram.
R = 0.287 kJ/kg*K

$P_1 = 100\,kPa$ $\quad T_1 = 300\,K$ $\quad k = 1.4$ $\quad$ *Ideal gas

$P_2 = 900\,kPa$ $\quad R = 0.287 \frac{kJ}{kg \cdot K}$

1st law: $\delta q - \delta w = du$

2nd law: $T ds = \delta q$

Boundary work: $dw = P dv$

$\delta q_{rev} - P dv = du \rightarrow \delta q_{rev} = du + P dv$

$\delta q_{rev} = dh - v dP$ ①

Energy balance for steady flow:

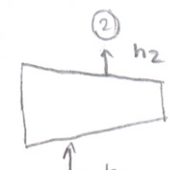
$\delta q_{rev} - \delta w_{rev} = dh$ ②

① into ②:

$\delta w_{rev} = -v dP$

$w_{rev} = -\int_1^2 v dP$ $\rightarrow$ for work input

$w_{rev} = \int_1^2 v dP$



$w_{out} = h_2 - h_1 = \Delta$

For ideal gas: $v = \frac{RT}{P}$ $\quad w_{rev} = R \int_1^2 T \frac{dP}{P}$

For isothermal process: $w_{rev} = RT \ln \frac{P_2}{P_1} \rightarrow \boxed{w_{rev} = 189.2\ kJ/kg}$

↑ isothermal

For isentropic process:

$w = \Delta h = C_p \Delta T$ $\quad$ where $k = C_p / C_v$

$C_p = C_v + R$

$P_1 v_1^k = P_2 v_2^k$ $\quad v = \frac{RT}{P}$

$P_1^{1-k} T_1^k = P_2^{1-k} T_2^k$

$T_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1-k}{k}} T_1 = \left(\frac{100}{900}\right)^{\frac{1-1.4}{1.4}} (300$

$= \frac{C_p R}{R} \Delta T$

$= \frac{C_p R}{C_p - C_v} \Delta T$

$= \frac{C_p/C_v\, R}{C_p/C_v - C_v/C_v} \Delta T = \frac{kR}{k-1} \Delta T = \frac{(1.4)(0.287 \frac{kJ}{kg \cdot K})(562 - 300K)}{1.4 - 1}$

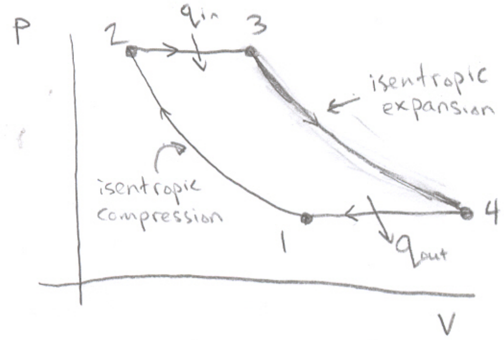
$\boxed{w_{rev} = 263.2 \frac{kJ}{kg}}$ ← isentropic

Name: ______________________
SID: ______________________

Question 4: [25 points]

A gas-turbine power plant is to produce 800 kW of power by compressing atmospheric air at 20 degrees C to 800 kPa. The maximum temperature is 800 degrees C.

a) draw this cycle on a PV diagram labeling the heat transfer and isentropic processes ✓
b) calculate the thermal efficiency
c) calculate Q_{in} ✓
d) calculate the temperature before the combustor ✓
e) calculate the required mass flow rate of air ✓



Cp = 1.004 kJ/kg*K
Cv = 0.717 kJ/kg*K
k = 1.4
R = 0.287 kJ/kg*K

$W_{out} = 800$ kW

$P_1 = 100$ kPa $T_1 = 293$ K
$P_2 = 800$ kPa $T_2 = ?$
$P_3 = 800$ kPa $T_3 = 1073$ K
$P_4 = 100$ kPa $T_4 = ?$

$$T_2 = T_1\left(\frac{P_1}{P_2}\right)^{\frac{1-k}{k}} = (293\text{ K})\left(\frac{100}{800}\right)^{\frac{1-1.4}{1.4}} \rightarrow \boxed{T_2 = 531\text{ K}}$$ ← Part d

$$q_{in} = C_p(T_3 - T_2) = \left(1.004\ \frac{kJ}{kg \cdot K}\right)(1073 - 531\text{ K}) \rightarrow \boxed{\begin{array}{l} q_{in} = 544\ \frac{kJ}{kg} \\ \dot{Q}_{in} = \dot{m} q_{in} = 1783.6\text{ kW} \end{array}}$$ ← Part c

$$T_4 = T_3\left(\frac{P_3}{P_4}\right)^{\frac{1-k}{k}} = (1073\text{ K})\left(\frac{800}{100}\right)^{\frac{1-1.4}{1.4}} \rightarrow T_4 = 592\text{ K}$$

$$q_{out} = C_p(T_4 - T_1) = \left(1.004\ \frac{kJ}{kg \cdot K}\right)(592 - 293\text{ K}) \Rightarrow q_{out} = 300\ \frac{kJ}{kg}$$

$$\dot{W}_{out} = \dot{m}(q_{in} - q_{out}) \rightarrow \dot{m} = \frac{\dot{W}_{out}}{q_{in} - q_{out}} = \frac{800\ kJ/s}{544 - 300\ \frac{kJ}{kg}} \rightarrow \boxed{\dot{m} = 3.28\ \frac{kg}{s}}$$ ← Part e

$$\eta_{th} = \frac{\dot{W}_{out}}{\dot{Q}_{in}} = \frac{800\text{ kW}}{1787\text{ kW}} \rightarrow \boxed{\eta_{th} = 0.4485}$$

Alternatively: $\eta_{th} = 1 - \frac{1}{r_p^{(k-1)/k}} = 1 - \frac{1}{8^{(1.4-1)/1.4}} \Rightarrow \boxed{\eta_{th} = 0.4485}$