EECS 20. Final Exam Solution 9 December 1998

1. (a) Let $\rho = \exp(2i \times 5\pi/12)$. Then

$$\sum_{k=0}^{n} \exp(2ik \times 5\pi/12) = \sum_{k=0}^{n} \rho^k = \frac{1-\rho^{n+1}}{1-\rho}$$

So this sum is zero if

$$1-\rho^{n+1} = 0 \Leftrightarrow \rho^{n+1} = \exp[2(n+1)\times 5\pi/12] = 1 \Leftrightarrow 2(n+1)\times 5\pi/12 \text{ is a multiple of } 2\pi$$

The smallest $n > 0$ for which this holds is $n = 11$.

(b) $Re[(1+i)\exp i\theta] = \cos\theta - \sin\theta = 0$ if $\theta = \pi/4$.

(c) Let $A = \alpha \exp i\theta$ in polar coordinates. Then

$$\begin{aligned} A\exp(i\omega t) + A^* \exp(-i\omega t) &= 2Re[\alpha \exp(i\omega t + \theta) \\ &= 2\alpha \cos(\omega t + \theta) = \cos(\omega t + \pi/4) \end{aligned}$$

if $\alpha = 1/2, \theta = \pi/4$. So $A = 1/2 \exp(i\pi/4)$.

2. The sets are shown in Figure 2

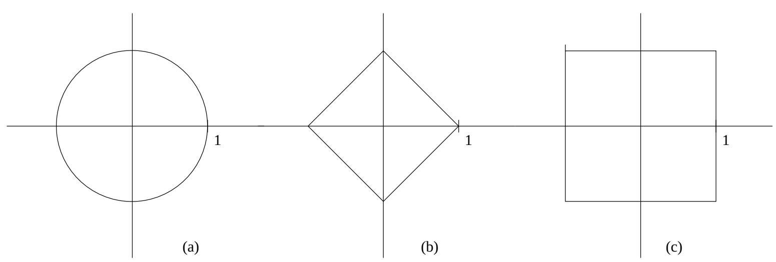

Figure 1: These are the required sets.

3. (a) The lists are

$$\begin{aligned} F_0 &= \{(a,a),(a,b),(b,c),(c,c),(c,d),(d,b)\} \\ F_1 &= \{(a,a),(b,b),(b,c),(d,a),(d,b),(d,d)\} \\ F_{01} &= \{(a,a),(b,c),(d,b)\} \\ F_{0or1} &= \{(a,a),(a,b),(b,b),(b,c),(c,c),(c,d),(d,a),(d,b),(d,d)\} \end{aligned}$$

(b) Both assertions are true.

(c) We have

$$F_{00} = \{(a,a),(a,b),(a,c),(b,c),(b,d),(c,c),(c,d),(c,b),(d,c)\}$$

4. Observe that $x'(t) = x(t) - x(t-1)$. By linearity and time invariance, it must therefore be true that

$$y'(t) = y(t) - y(t-1) = \begin{cases} 1, & \text{if } 0 \le t < 1 \\ -1, & \text{if } 2 \le t < 3 \\ 0, & \text{otherwise} \end{cases}$$

This is sketched below:

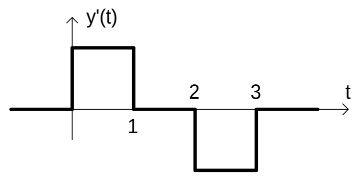

Figure 2: The output signal.

5. (a) Observe that $x(n) = cos(\pi n/2) = (e^{j\pi n/2} + e^{-j\pi n/2})/2$. Since $H(\pi/2) = H(-\pi/2) = \pi/2$, it follows that $y(n) = \frac{\pi}{2} cos(\pi n/2)$.

   (b) Observe that $x(n) = 5e^{j0n}$. Since $H(0) = 0$, it follows that $y(n) = 0$.

   (c) Observe that $x(n) = cos(\pi n) = (e^{j\pi n} + e^{-j\pi n})/2$. Since $H(\pi) = H(-\pi) = \pi$, it follows that

$$y(n) = \pi cos(\pi n) = \begin{cases} +\pi, & n \text{ even} \\ -\pi, & n \text{ odd} \end{cases}$$

6. The state transition diagram is shown in Figure 3. Note the names of the four states indicate the pattern that is remembered. The output sequence is $ffffffftfft \cdots$.

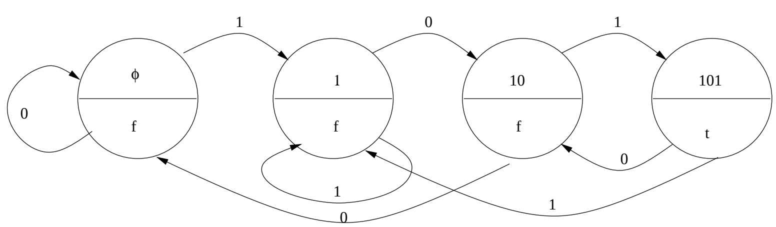

Figure 3: Required state transition diagram.

7. (a) The general expression is $\forall t = 0, 1, \cdots$

$$y(t) = c'A^t x_0 + \sum_{s=0}^{t-1} c'A^s b u(t-s-1)$$

   (b) We have $\forall t$

$$1 = c'A^t x_0 + \sum_{s=0}^{t-1} c'A^s b \quad (1)$$

$$4 = 2c'A^t x_0 + \sum_{s=0}^{t-1} c'A^s b \tag{2}$$

$$\tag{3}$$

i. If we subtract (1) from (2) we get $\forall t$

$$3 = c'A^t x_0 \tag{4}$$

so the zero-input response is $\forall t,\ y(t) = 3$.

ii. If we subtract (4) from (1) we get

$$-2 = \sum_{s=0}^{t-s-1} c'A^s b$$

So the zero-state response is $\forall t,\ y(t) = -2$.