EECS20, Spring 2002 – Solutions to Final Exam

0. **Typos/Corrections -** Announced during exam.

   -All £ symbols stand for the letters *f i*.

   -For Problem 2, assume the system is causal.

   -Problem 4 is for 20 points, not 10 points.

   -In 4(c), the argument of y should be τ, not "t".

   -In 4(d), take the input to be $x(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-3)$.

1. **20 points**

   (a) By inspection, we obtain that $w_0 = \pi/2$.

   $A_1 = A_2 = A_6 = A_{10} = 1$, $A_k = 0$ otherwise.

   $\phi_k = 0$ for all k.

   (b) Using the relations

$$
\begin{aligned}
X_0 &= A_0, \\
X_k &= 0.5 A_k e^{i\phi_k}, \quad k = 1, 2, \cdots \\
X_{-k} &= X_k{}^* = 0.5 A_k e^{-i\phi_k}, \quad k = 1, 2, \cdots
\end{aligned}
$$

   we get

$$
X_k = \begin{cases} 0.5, & k = \pm 1, \pm 2, \pm 6, \pm 10 \\ 0, & \text{otherwise} \end{cases}
$$

   (c)

$$
\begin{aligned}
y(t) &= \sum_{k=-\infty}^{\infty} X_k H(w_0 k) e^{ikw_0 t} \\
&= X_2 H(\pi) e^{i\pi t} + X_{-2} H(-\pi) e^{-i\pi t} \\
&= -\cos(\pi t).
\end{aligned}
$$

2. **15 points**

(a)

$$s(n) = \begin{bmatrix} y(n-1) \\ y(n-2) \\ x(n-1) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & \alpha^2 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 0. \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad c = \begin{bmatrix} 0 & -\alpha^2 & 2 \end{bmatrix},$$

$d = 1.$

(b) Substituting $x(n) = \delta(n)$, we can compute $y(n)$ directly from the relation

$$\forall n \in Integers, \;\; y(n) + \alpha^2 y(n-2) = x(n) + 2\,x(n-1).$$

Assuming causality, we have that $y(n) = 0$ for $n < 0$. We then obtain

$y(0) = 1,$

$y(1) = 2,$

$y(2) = \alpha^2,$

$y(3) = -2\alpha^2,$

$y(4) = \alpha^4,$

$y(5) = 2\alpha^4,$

$y(6) = -\alpha^6,$

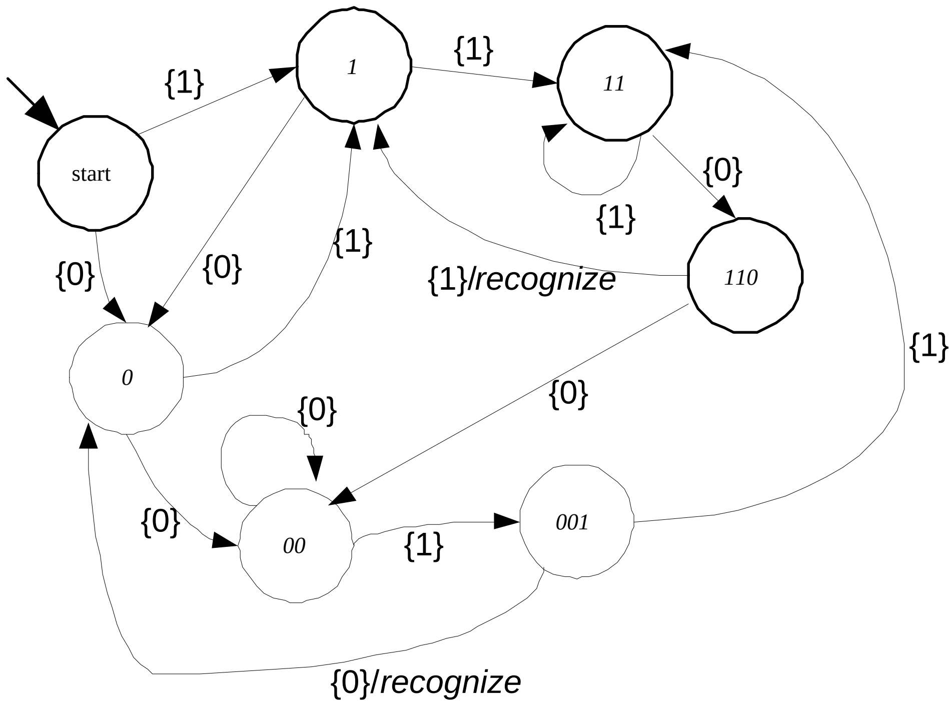
$y(7) = -2\alpha^6.$

A generic expression is

$$y(n) = \begin{cases} (-1)^{n/2}\alpha^n, & \text{if n is even, } n/ge0 \\ (-1)^{(n-1)/2} 2\alpha^{n-1}, & \text{if n is odd, } n/ge0 \\ 0, & n < 0 \end{cases}$$

(c) The system is stable for $|\alpha| < 1$.

3. **15 points** The following state machine implements *CodeRecognizer*.



4. **20 points**

(a) $X(\omega) = X(\omega + 2\pi)$. Hence, $p = 2\pi$ and $\omega_0 = 2\pi/p = 1$.

(b) $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-i\omega n} = \sum_{n=-\infty}^{\infty} x(-n)e^{i\omega n}$. By inspection, we obtain $\tilde{X}_n = x(-n)$.

(c) $y(\tau)$ is the CTFT of $X(\omega)$. The inverse-CTFT relationship is

$$X(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} y(\tau)e^{i\tau\omega}\, d\tau.$$

We also have that

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(-n)e^{in\omega}.$$

The two expressions for $X(\omega)$ can be equal if and only if

$$y(\tau) = 2\pi \sum_{n=-\infty}^{\infty} x(-n)\delta(n-\tau).$$

(d) When $x(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-3)$, $y(\tau) = 2\pi\delta(\tau) + 4\pi\delta(\tau + 2) + 6\pi\delta(\tau + 3)$. Hence, the plot of $y(\tau)$ consists of three spikes of amplitudes 2π, 4π and 6π, at $\tau = 0$, $\tau = -2$ and $\tau = -3$, respectively.

5. **10 points**

(a) Using the relationship $H(\omega) = \sum h(n)e^{i\omega n}$, we obtain that

$$H_1(\omega) = 1 + e^{i\omega} + e^{i\omega 2} + e^{i\omega 3} + e^{i\omega 4},$$

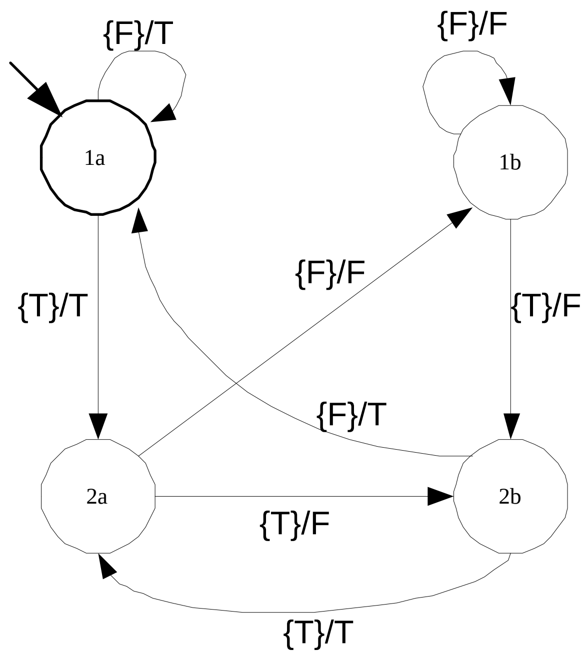
$$H_2(\omega) = 1 - e^{i\omega} + e^{i\omega 2} - e^{i\omega 3} + e^{i\omega 4}.$$

(b) From part (a), and using the fact that $e^{i\pi} = -1$, we see that $H_1(\omega) = H_2(\omega + \pi)$. So $\phi = \pi$ works. In fact, ϕ can be chosen to be any odd, integer multiple of π.

6. **10 points**

(a) (4 pts) Yes, the feedback composition has a unique non-stuttering input for all reachable states.

(b) (6 pts) Yes. The following is a state transition diagram for the composite machine:



7. **10 points**

(a) L, NC

(b) TI, C

(c) L, NC