MIDTERM SOLUTION – ME104 SEC. 2 November 17, 2005

Department of Mechanical Engineering

University of California, Berkeley

Closed Book (one sheet of formulae provided), 80 minutes

Each problem is worth 20 points

1. From $r_{min} = a(1-e)$, $2R = 6R(1-e)$

$$e = \frac{2}{3}$$

Check: $b = a\sqrt{1-e^2} = 6R\sqrt{1-\left(\frac{2}{3}\right)^2} = 2\sqrt{5}\,R$ ✓

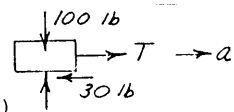
At B, $r = \sqrt{(4R)^2 + (2\sqrt{5}R)^2} = 6R$

$$v_B^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right) = 2gR^2\left(\frac{1}{6R} - \frac{1}{12R}\right)$$

$$v_B = 0.408\sqrt{gR} = 0.408\sqrt{9.825\,(6371\cdot 10^3)}$$

$$= \underline{3230\ m/s}$$

2.



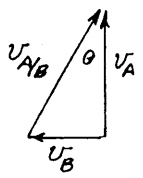
$F = \mu_k N = 0.30(100) = 30$ lb

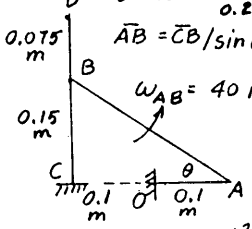
$a = a_0 + a_{rel} = 2a_0 = 2(6) = 12\ ft/sec^2$

$\Sigma F = ma$; $T - 30 = \frac{100}{32.2}(12)$, $\underline{T = 67.3\ lb}$

3. $v_A = v_B + v_{A/B}$

$\theta = \tan^{-1}\frac{0.15}{0.2} = \tan^{-1}\frac{3}{4}$





$\bar{AB} = \bar{CB}/\sin\theta = \frac{0.15}{3/5} = 0.25$ m

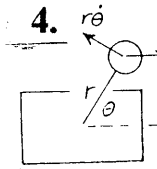
$\omega_{AB} = 40$ rad/s

$v_{A/B} = \bar{AB}\,\omega_{AB} = 0.25(40) = 10$ m/s

$v_B = v_{A/B}\sin\theta = 10\left(\frac{3}{5}\right) = 6$ m/s

$v_D = \frac{\bar{DC}}{\bar{BC}}v_B = \frac{0.225}{0.15}6 = \underline{9\ m/s}$

4.



$\Sigma F_x = 0$ for system so $\Delta G_x = 0$

$(G_x)_{\theta=0} = (20+5)(0.6) = 15.0$ N·s

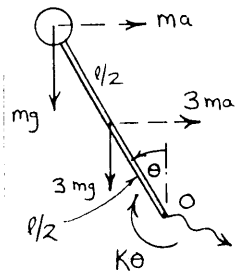
$(G_x)_{\theta=60°} = (20+5)v - 5(1.6)\sin 60°$

$= 25v - 6.93$ N·s

$r\dot{\theta} = 0.4(4) = 1.6$ m/s

Thus $15.0 = 25v - 6.93$, $v = 21.9/25 = \underline{0.88\ m/s}$

5.



$\curvearrowright \Sigma M_0 = \Sigma ma_d$: $K\theta - 3mg\left(\frac{\ell}{2}\sin\theta\right)$

$- mg(\ell\sin\theta) = 3ma\left(\frac{\ell}{2}\cos\theta\right)$

$+ ma(\ell\cos\theta)$

Simplify to

$K\theta - \frac{5}{2}mg\ell\sin\theta = \frac{5}{2}ma\ell\cos\theta$

With $m = 0.5$ kg, $\ell = 0.6$ m, $a = 2g$, and $\theta = 20°$, K is found to be $\underline{K = 46.8\ \frac{N\cdot m}{rad}}$