EECS20, Spring 2002 – Solutions to Midterm 2

1. **30 points**

   (a) By inspection, we obtain that $w_0 = 1$, $A_0 = 3$, $A_3 = 2$, $A_4 = 3$, $\phi_3 = -\pi/2$. All other A_k and ϕ_k are equal to zero.

   (b) Using the relations

$$\begin{aligned} X_0 &= A_0, \\ X_k &= 0.5A_k e^{i\phi_k}, \quad k = 1, 2, \cdots \\ X_{-k} &= {X_k}^* = 0.5A_k e^{-i\phi_k}, \quad k = 1, 2, \cdots \end{aligned}$$

   we get

$$X_k = \begin{cases} 3, & k = 0 \\ -i, & k = 3 \\ i, & k = -3 \\ 1.5, & k = -4, 4 \\ 0, & \text{otherwise} \end{cases}$$

   (c)

$$\begin{aligned} y(t) &= \sum_{k=-\infty}^{\infty} X_k H(w_0 k) e^{ikw_0 t} \\ &= X_0 H(0) + X_3 H(3) e^{i3t} + X_{-3} H(-3) e^{-i3t} + X_4 H(4) e^{i4t} + X_{-4} H(-4) e^{-i4t} \\ &= 4\sin(3t) + 6\cos(4t). \end{aligned}$$

2. **20 points**

   (a)

$$s(n) = \begin{bmatrix} x(n-1) \\ x(n-2) \\ y(n-1) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 1.1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} 0 & 2 & 1.1 \end{bmatrix}, \quad d = 0.$$

   (b) Given that $x(n) = \delta(n)$, we can compute $y(n)$ directly from the relation $y(n) = 2x(n-2) + 1.1y(n-1)$. We get

$$y(n) = \begin{cases} 2(1.1)^{n-2}, & n \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

   (c) No, this system is *not* stable. From part (b) above, we see that for one particular input ($x(n) = \delta(n)$), the output $y(n) \to \infty$ as $n \to \infty$.

3. **20 points**

   (a) TI

   (b) TI

   (c) N

   (d) TI

   (e) L

4. **15 points**

   The new input (call it $x_2(t)$) can be expressed in terms of the old input (call it $x_1(t)$) as $x_2(t) = x_1(t) - x_1(t-1)$. Using the linearity and time-invariance properties, we obtain that $y_2(t) = y_1(t) - y_1(t-1)$, which can be simplified to

$$y_2(t) = \begin{cases} \sin(\pi t), & 0 \leq t < 4 \\ 0, & \text{otherwise} \end{cases}$$