Mechanical Engineering Department
University of California at Berkeley

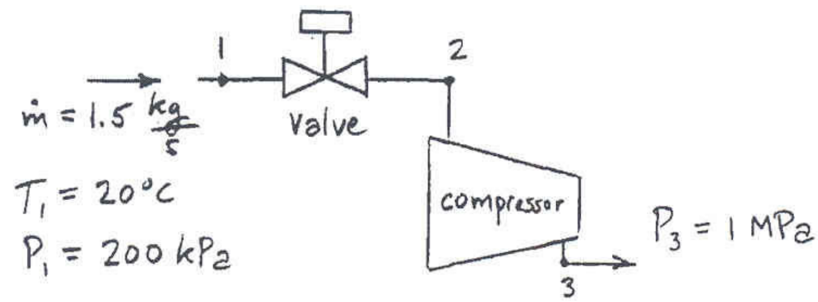
ME105B, Fall 2004
Prof. V. Carey

Midterm I

Name w/soln

Instructions: Do both problems. Show all work and make sure that your final answers are clearly distinguishable *with proper units.*

1. (50 points)



The valve and compressor shown in the schematic above are part of a natural gas supply system. Methane gas flows through the valve and compressor at a steady rate of 1.5 kg/s. The methane supply line (at 1) is maintained at a temperature of 20 °C and a pressure of 200 kPa. The compressor exhaust pressure is fixed at 1.0 MPa and the compressor operates adiabatically.

In your analysis treat the methane as an ideal gas with constant specific heats (c_p = 2.254 kJ/kgK, c_v = 0.6179 kJ/kgK).

(a) For the wide-open valve conditions, the pressure at 2 is the same as at 1. Determine the minimum possible power input to the compressor that must be supplied to raise the gas pressure to 1 MPa under these conditions.

(b) For the valve partially open, the pressure at 2 is 120 kPa. For these conditions, determine the minimum possible power input to the compressor that must be supplied to raise the gas pressure to 1 MPa.

(c) Determine the overall change in entropy per kg of the methane $s_3 - s_1$ for the wide-open valve conditions in part (a), and for the partially open conditions in part (b).

(a) $P_2 = P_1,\ T_2 = T_1$ SSSF, rev & adiab, 2nd law $\Rightarrow s_3 = s_2$

const s, const cp, ideal gas $\rightarrow \dfrac{T_3}{T_2} = \left(\dfrac{P_3}{P_2}\right)^{(k-1)/k}$

$k = c_p/c_v = 2.254/\overset{(1.735)}{.6179} = \overset{(1.299)}{3.648}$

(a) cont'd

$$T_3 = 293.2\left(\frac{1000}{200}\right)^{.726\ (.230)} = 942.6\ (424.3)$$

1ST Law: $\dot{W} = \dot{m}(h_2 - h_3) = \dot{m}c_p(T_2 - T_3) = 1.5(2.254)(293.2 - 942.6\ (424.3))$

$$= -\underline{2196\ (444)}\ kW$$

(b) SSSF, adiabatic $\Rightarrow h_2 = h_1 \Rightarrow T_2 = T_1$ for ideal gas, $P_2 = 120$ kPa

$$T_3 = 293.2\left(\frac{1000}{120}\right)^{.726\ (.230)} = 1365.8\ (477.2)$$

$$\dot{W} = 1.5(2.254)(293.2 - 1365.8\ (477.2)) = \underline{-3627\ (622)\ kW}$$

(c) <u>open</u> $s_3 - s_1 = c_p \ln(T_3/T_1) - R\ln(P_3/P_1)$

$$R = c_p - c_v = 2.254 - .6179\ (1.735) = 1.636\ (.5190)\ kJ/kg\,K$$

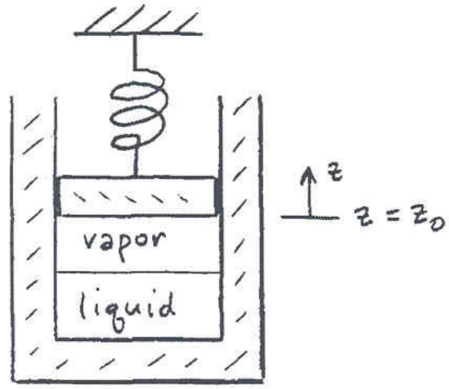
$$s_3 - s_1 = 2.254\ln(942.6\ (424.3)/293.2) - 1.636\ (.5190)\ln(1000/200)$$

$$= 0.0006896\ (.0000713) = \underline{0.00}\ (0.00)\ kJ/kg\,K$$

<u>partially open</u>

$$s_3 - s_1 = 2.254\ln(1365.8\ (477.2)/293.2) - 1.636\ (.5190)\ln(1000/200)$$

$$= \underline{0.8366}\ (.2641)\ kJ/kg\,K$$



Initial system pressure = P_0 = 200 kPa

2. (50 points) The piston and cylinder device above contains vapor and liquid water. The total mass of water in the system is 2.5 g. The area of the piston A_p is 0.01 m^2 and the spring constant k_s is 200,000 N/m. If the piston is displaced upward from its initial position ($z = z_0$) to a new position z, the force acting downward on the piston is $P_0A_p + k_s(z - z_0)$. Initially the pressure inside is 200 kPa, the quality is 0.1, and the force exerted by the spring is zero.

The system undergoes a process in which 1200 J of heat are input and the piston moves up to a position 3 cm above its original position ($z_2 - z_0 = 0.03$ m). Determine the work interaction with the spring, and the temperature and mass of vapor in the system at the new state.

$$W = \int F\,dz = \int_{z_0}^{z_0+0.03} [P_0 A_p + k_s(z-z_0)]\,dz = \left[P_0 A_p (z-z_0) + \tfrac{1}{2}k_s(z-z_0)^2\right]_{z_0}^{z_0+.03}$$

$$= P_0 A_p (.03) + \tfrac{1}{2} k_s (.03)^2 = (2\times10^5)(.01)(.03) + \tfrac{1}{2}(2\times10^5)(.03)^2$$

$$W = 60 + 90 = 150\ J$$

spring work = $\underline{90\ J}$

$$\Delta U = Q - W \rightarrow u_2 = u_0 + \frac{Q-W}{m} = u_0 + \frac{1200-150}{.0025(1000)} = u_0 + 420$$

@ 200 kPa, $u_f = 504.5$ kJ/kg, $u_{fg} = 2025.0$

$$u_0 = u_f + x_0 u_{fg} = 504.5 + 0.1(2025.0) = 707.0\ kJ/kg.$$

$$u_2 = 707.0 + 420 = 1127\ kJ/kg$$

$$P_2 = \frac{F_2}{A_p} = P_0 + \frac{k_s(z_2-z_0)}{A_p} = 200 + \frac{2\times10^5(.03)}{.01(1000)} = 800\ kPa.$$

@ $P_2 = 800$ kPa $u_f = 720.2$, $u_{fg} = 1856.6$ kJ/kg

$u_2 = u_f + x_2 u_{fg} \rightarrow x_2 = \frac{u_2 - u_f}{u_{fg}} = \frac{1127 - 720.}{1857} = 0.219$

mass of vapor at state 2 $= m x_2 = 2.5(.219) = \underline{0.55\ g}$

$T_2 = T_{sat}(P_2) = \underline{170.4\ °C}$

2.18g

=.848

ALTERNATE SOLUTION:

@ 200 kPa, $v_f = .00106$, $v_g = .8857$, $v_{fg} = .8846$

$v_0 = v_f + x_0 v_{fg} = .00106 + .1(.8846) = .08952\ m^3/kg$

syst. vol $= V_0 = m v_0 = .0025(.08952) = 2.238 \times 10^{-4}\ m^3$

new vol $= V_0 + \Delta z A_p = 2.238 \times 10^{-4}\ (.03)(.01) = 5.238 \times 10^{-4}$

$v_2 = V_2 / m = 5.238 \times 10^{-4} / .0025 = .20952$

@ 800 kPa, $v_f = .00112$, $v_g = .2404$

$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{.20952 - .00112}{.2404 - .00112} = 0.871$

mass of vapor at state 2 $= m x_2 = 2.5(.871) = \underline{2.18\ g}$

$T_2 = T_{sat}(P_2) = \underline{170.4\ °C}$

$\underline{Q = 4225}$

$u_2 = (720.2) + .871(1856.6) = 2337.1$ kJ/kg

$u_2 = 2337.1 - 707 = \frac{Q - W}{m(1000)} =$

2337. $(1630\ kJ/kg)(.0025 kg) + W = Q$

4.075 kJ + 150 J = Q

4075 + 150 = 4225.5