EECS 20. Final Exam
December 19, 2002.

Please use these sheets for your answer. **Write clearly and show your work.** Please check that you have 12 numbered pages.

Print your name and lab time below

Name: ______________________________

Lab time: ___________________________

Problem 1 (12):

Problem 2 (16):

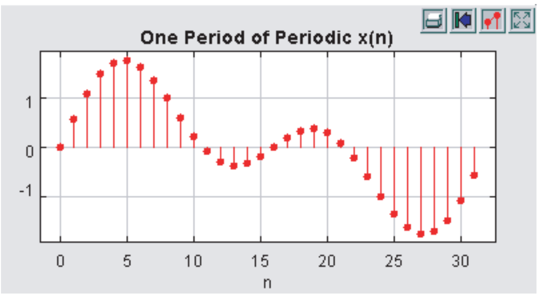
Problem 3 (28):

Problem 4 (24):

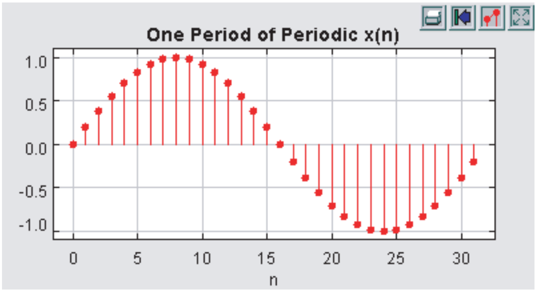
Problem 5 (20):

Total:

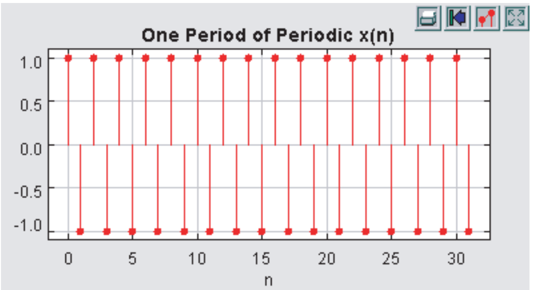
1. **12 points.** Suppose that x is a discrete-time signal with period $p = 32$. Below are plotted six possible such signals x. For each of these, match one of the six plots on the next page, or indicate that none matches.



matching FS:________________________



matching FS:________________________



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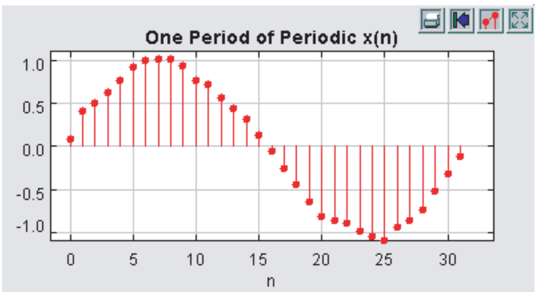
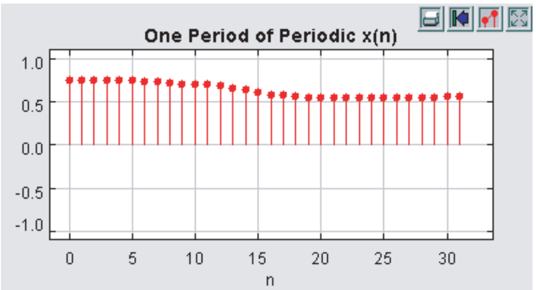


matching FS:________________________



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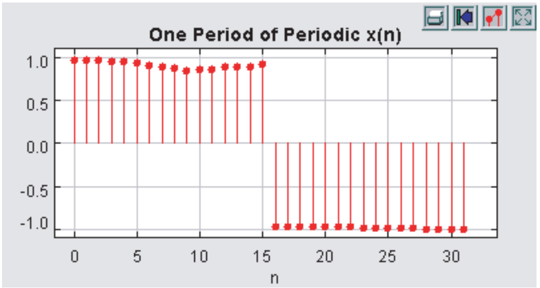
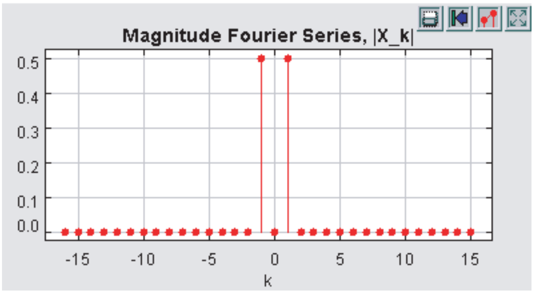
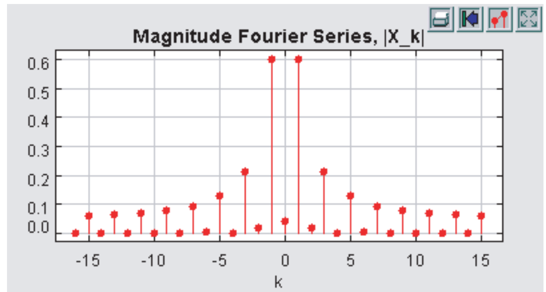


matching FS:________________________

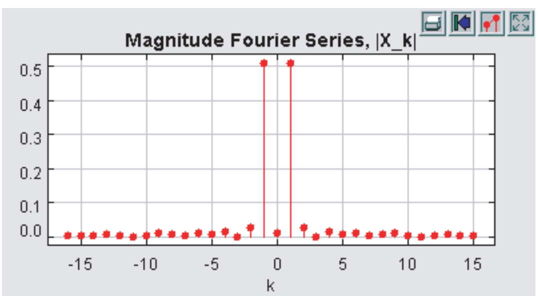
Below are six plots of the magnitudes $|X_k|$ of the Fourier series coefficients X_k of a periodic signal x with period $p = 32$, for six different such periodic signals.



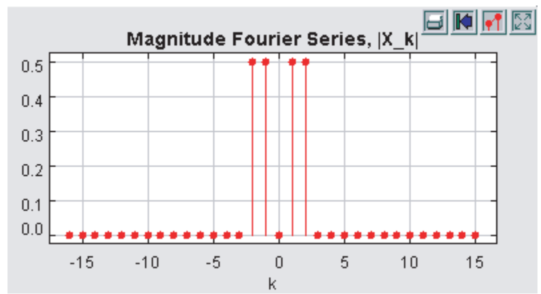
(a)



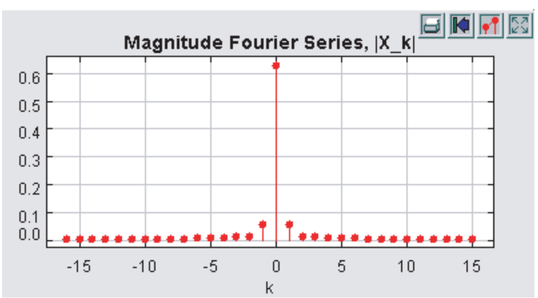
(b)



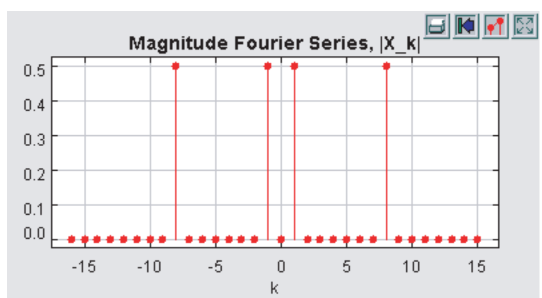
(c)



(d)



(e)



(f)

2. **16 points.** Let $B = \{t, f\}$ be the set of binary truth values. Consider a nondeterministic state machine with

$$\begin{aligned} \textit{States} &= B \times B \\ \textit{Inputs} &= B \cup \{\textit{absent}\} \\ \textit{Outputs} &= B \cup \{\textit{absent}\} \\ \textit{initialState} &= (f, f) \end{aligned}$$

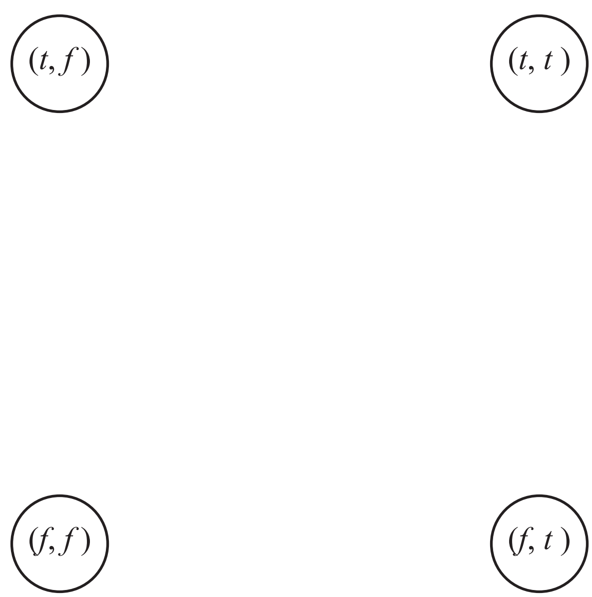
and the *possibleUpdates* is given by $\forall\ (s_1, s_2) \in \textit{States}$ and $i \in \textit{Inputs}$,

$$\textit{possibleUpdates}((s_1, s_2), i) = \begin{cases} \{((s_1, s_2), \textit{absent})\} & \text{if } i = \textit{absent} \\ \{((s_1, s_2), s_1), ((s_1, \neg s_2), s_1)\} & \text{if } i = f \\ \{((\neg s_1, s_2), s_1), ((\neg s_1, \neg s_2), s_1)\} & \text{if } i = t \end{cases}$$

where the notation $\neg$ means the following:

$$\forall\ b \in B, \quad \neg b = \begin{cases} f & \text{if } b = t \\ t & \text{if } b = f \end{cases}$$

(a) 6 points. Draw the state transition diagram below.



(b) 4 points. For the input sequence $(f, f, t, t, f, f, \cdots)$, give one possible output sequence. You need show only the first six output symbols.

(c) 6 points. Give a simpler deterministic state machine that is bisimilar to this one, and give the bisimilarity relation.

3. **28 points.** Determine whether each of the following statements is true or false. There will be no partial credit, so consider your answer carefully.

(a) $\forall\, t \in \textit{Reals}, \quad \cos(\pi t) = (-1)^t$.

(b) If the input to a continuous-time LTI system is x such that

$$\forall\, t \in \textit{Reals}, \quad x(t) = \cos(t) + \sin(t),$$

then $\exists\, A \in$ *Reals* and $\exists\, \phi \in$ *Reals* such that the output y of the LTI system is

$$y(t) = A\cos(t + \phi).$$

(c) $[\{1, 2\} \to \{1, 2\}] \subset [\{1, 2\} \to \textit{Naturals}]$.

(d) Suppose $[A, b, c, d]$ and $[A', b', c', d']$ describe two different LTI systems. If the frequency response of the two is the same, then it must be that $A = A'$.

(e) An identity function is $f\colon X \to X$ where $\forall\, x \in X, \quad f(x) = x$. Suppose that $f\colon X \to X$ and $g\colon Y \to Y$ are both identity functions, and further that $X \subset Y$. Then *graph*$(f) \subset$ *graph*(g).

(f) For any sets X and Y, $X \in P(X \cup Y)$, where P denotes the powerset.

(g) If y is a continuous-time signal given by

$$\forall\, t \in \textit{Reals}, \quad y(t) = \sum_{k=-\infty}^{\infty} \delta(t-k),$$

where δ is the Dirac delta function, then $\forall\, t \in$ *Reals* where $t \notin$ *Integers*, $y(t) = 0$.

4. **24 points.** Consider a discrete-time function x where

$$\forall\, n \in \textit{Integers}, \quad x(n) = 1 + \cos(\pi n/4) + \sin(\pi n/2).$$

(a) Find the period p and fundamental frequency ω_0. Give the units.

(b) Find K and the Fourier series coefficients $A_0, \cdots, A_K$ and $\phi_1, \cdots, \phi_K$ in

$$x(n) = A_0 + \sum_{k=1}^{K} A_k \cos(k\omega_0 n + \phi_k).$$

(c) Find the Fourier series coefficients X_k in

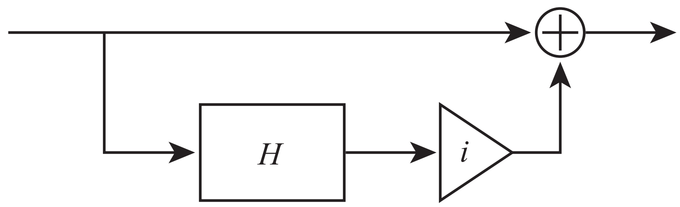
$$x(n) = \sum_{k=0}^{p-1} X_k e^{ik\omega_0 n}.$$

(d) Let x be the input to an LTI system with frequency response H given by

$$\forall\, \omega \in \textit{Reals}, \quad H(\omega) = \begin{cases} -i & \text{if } 0 < \omega < \pi \\ 0 & \text{if } \omega = 0 \text{ or } \omega = \pi \\ i & \text{if } -\pi < \omega < 0 \end{cases}$$

For other values of ω, $H(\omega)$ is determined by the periodicity of H. (Such a system is called a Hilbert filter.) Find the output.

(e) Find the frequency response H' of a new system constructed as follows,



where H is the Hilbert filter from part (d), and the triangle with an i scales its input by the imaginary number i. Note that even if the input is real-valued, the output is likely to be complex-valued. Such a system is called a Hilbert transformer; approximations to it are widely used in digital communication systems.

(f) Let the input to the system in part (e) be x. What is the output?

5. **20 points.** Let the continuous-time signal c given by

$$\forall\, t \in \textit{Reals}, \quad c(t) = 2\cos(\omega_c t)$$

be a carrier wave for a radio signal. Let x given by

$$\forall\, t \in \textit{Reals}, \quad x(t) = 2\cos(\omega_x t)$$

be the signal to be carried by that radio signal (that it, it is a highly simplified stand-in for, say, a voice signal). To be concrete, let $\omega_c = 2\pi \cdot 8000$ radians/second, and $\omega_x = 2\pi \cdot 400$ radians/second.

(a) Find and sketch the CTFT of y where

$$\forall\, t \in \textit{Reals}, \quad y(t) = c(t)x(t).$$

Label your sketch carefully. **Hint**: The CTFT of $e^{i\omega_0 t}$ is $2\pi\delta(\omega - \omega_0)$.

(b) Let y from part (a) be the input to an LTI system with frequency response H where

$$\forall\, \omega \in \textit{Reals}, \quad H(\omega) = \begin{cases} 0 & \text{if } \omega \leq 0 \\ 1 & \text{if } \omega > 0 \end{cases}$$

Find the output u.

(c) For the same u from part (b), let

$$u' = \textit{Sampler}_T(u),$$

where $T = 1/8000$ seconds. Find a simple expression for u'.

(d) Give the signal $z = \textit{IdealInterpolator}_T(u')$, where again $T = 1/8000$ seconds, and u' is from part (c).