EECS 20. Midterm No. 1
October 4, 2002.
Solution

1. **50 points.** Please indicate whether the following statements are true or false. There will be no partial credit. They are either true or false. So please be sure of your answer.

   (a) $\forall\, t \in \mathit{Reals}, \quad (t, t+1) \in \mathit{Reals}^2$ **true**

   (b) $\exists\, x \in \mathit{Integers}, \quad \{(x, x+1)\} \subset \{1,2,3\}^2$ **true**

   (c) If $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then $\exists\, x \in A$ such that $\forall y \in B$, $x <= y$. **true**

   (d) $P(A \cup B) = P(A) \cup P(B)$, where P denotes the power set. **false**

   (e) For any two functions $f\colon A \to A$ and $g\colon A \to A$, where A is a set, $f \circ g = g \circ f$. **false**

   (f) Let $f\colon \mathit{Reals} \to \mathit{Reals}$ be a function where $\forall x \in \mathit{Reals}$, $f(x) = x \sin(x)$. Then f is onto. **true**

   (g) For the same function f in the previous part, f is one-to-one. **false**

   (h) Let $A = [-1, 1]$. Consider a function f where $\forall\, x \in A$, $f(x) = x \sin(2\pi x)$. Then $f \in [A \to A]$. **true**

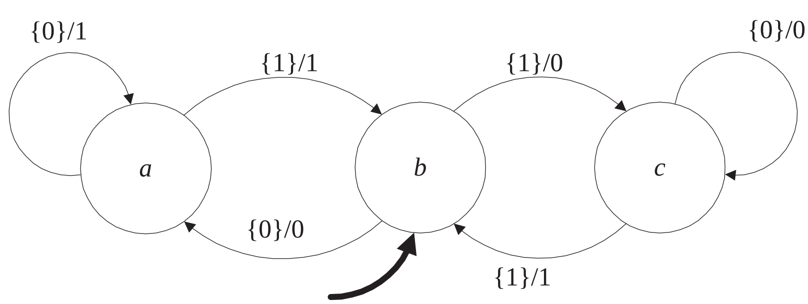
   (i) $[\{1,2,3\} \to \{1,2\}] \subset [\{1,2,3\} \to \mathit{Naturals}]$. **true**

   (j) $X \times Y \in \{g \mid g = \mathit{graph}(f) \wedge f\colon X \to Y\}$. **false**

   (k) Given two state machines A and B, if A simulates B and A is deterministic, then B simulates A. **true**

   (l) Consider two state machines A and B with state spaces States_A and States_B. If in each state machine, all states are reachable, then in the side-by-side composition, all states in $\mathit{States}_A \times \mathit{States}_B$ are reachable. **true**

2. **35 points.** Consider the state transition diagram shown below.



   Give each of the following:

   (a) *States* = $\{a, b, c\}$

   (b) *Inputs* = $\{0, 1, \mathit{absent}\}$

(c) *Outputs* = $\{0, 1, absent\}$

(d) Give the domain and range, and fill in the table for
update: *States* $\times$ *Inputs* $\rightarrow$ *States* $\times$ *Outputs*

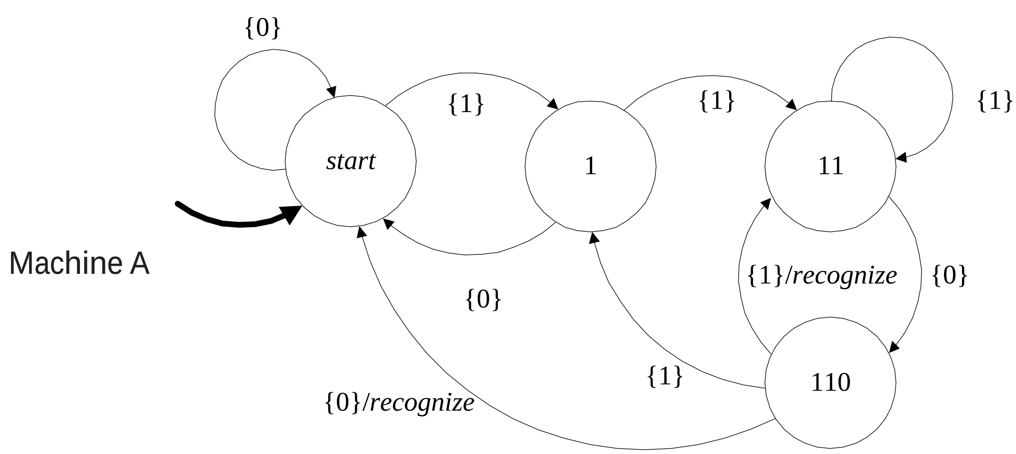
current state	(*next state, output symbol*) under specified input symbol		
	0	1	*absent*
a	$(a, 1)$	$(b, 1)$	$(a, absent)$
b	$(a, 0)$	$(c, 0)$	$(b, absent)$
c	$(c, 0)$	$(b, 1)$	$(c, absent)$

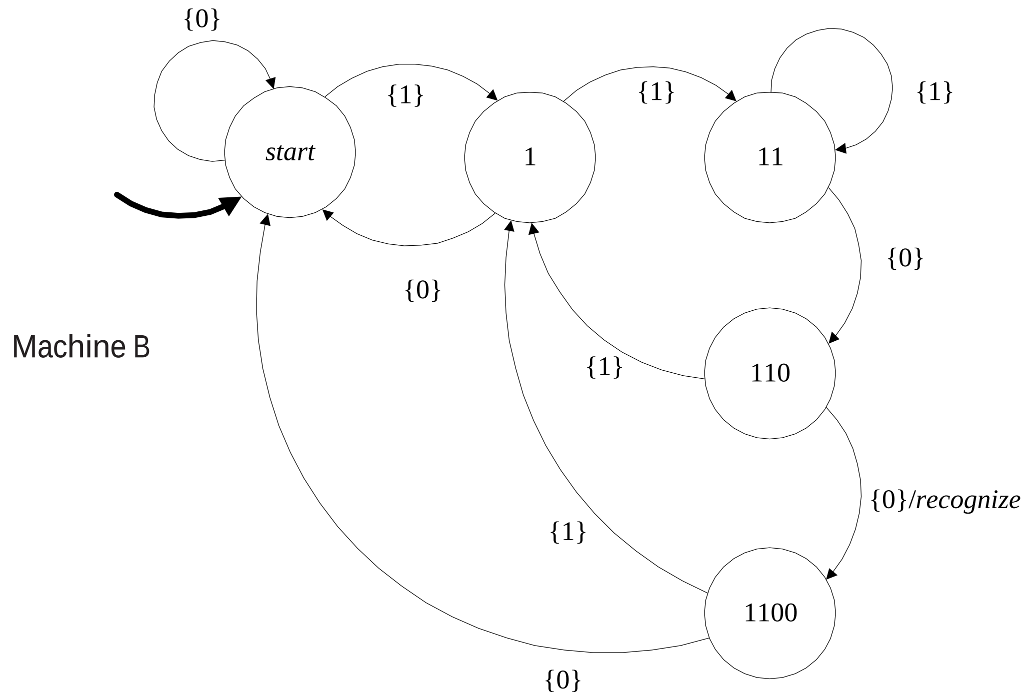
(e) *initialState* = b

(f) Compose this state machine in a feedback loop, where its output is connected to its input. Assume the output of the composition is the output of this state machine. Give the set *Behaviors* for the feedback composition. You may ignore stuttering reactions, and give only the behaviors with no stuttering reactions.

$$Behaviors = \{((react, react, react, react, ...), (0, 1, 0, 1, ...))\}$$

3. **15 points.** Consider the following two state machines:





These are similar to the machine *CodeRecognizer* studied in the text and in the homework. Determine whether A simulates B, B simulates A, neither, or both. Give the relevant simulation relations, if any.

A simulates B only.

$$\mathit{SimulationRelation} = \{(\mathit{start}, \mathit{start}), (1, 1), (11, 11), (110, 110), (1100, \mathit{start})\}.$$