EECS20, Spring 2002 – Solutions to Midterm 1

1. **15 points**

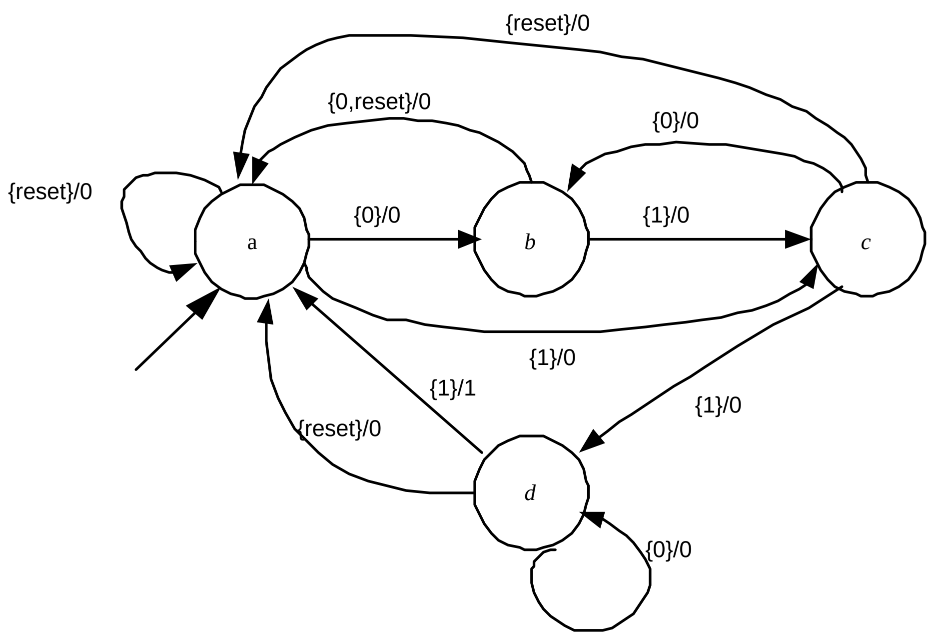
   (a) *Not well-formed*: The feedback composition has more than one £xed point solution when the initial state of B is 1 and of C is x.

   (b) *Well-formed*: The feedback composition has a unique non-stuttering input for all reachable states.

   (c) *Not well-formed*: The output of C is not a subset of the inputs of A.

2. **20 points**

   (a) (7 points) State transition diagram for A is shown below:



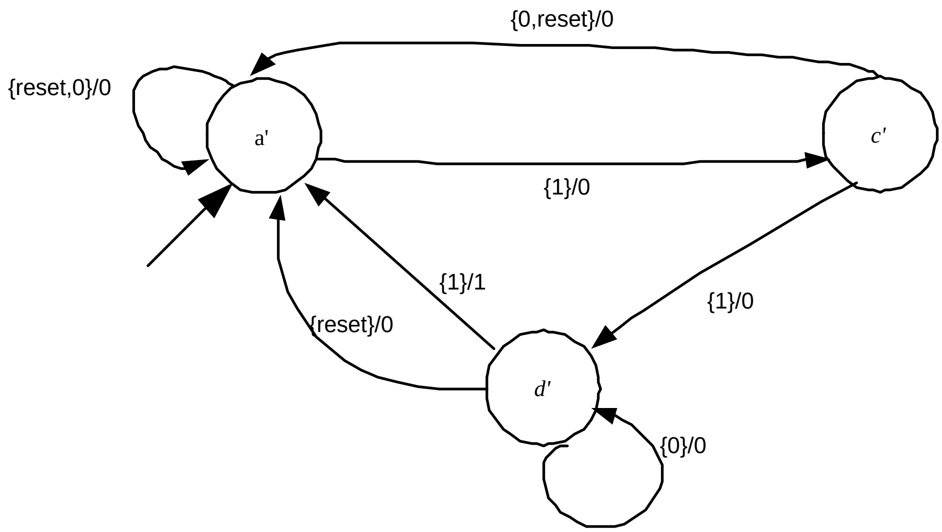
   (b) (5 points) The following machine is bisimilar to A:

   initialState=a

   Inputs = $\{$0,1,reset,absent$\}$

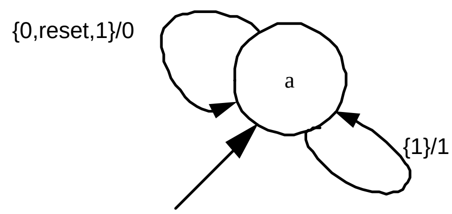
   Outputs=$\{$0,1,absent$\}$

   States=$\{$a’,c’,d’$\}$



(c) (3 points) $\{$*(a,a'),(b,a'),(c,c'),(d,d')*$\}$

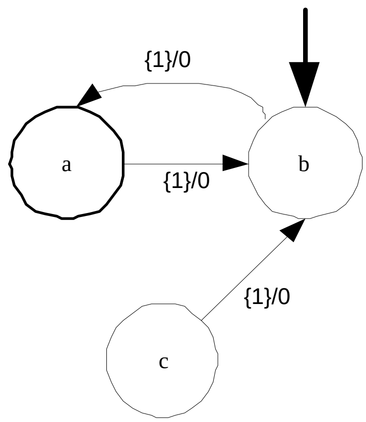
(d) (5 points)



3. **20 points**

(a) Yes, the machine B can be de£ned via a £nite state machine model. The set of *Inputs*, *Outputs* and *States*, and *initialState* are the same as in A; only the *update* function changes appropriately. (Note: It is possible to concoct examples where B stays in its initial state for all possible inputs; i.e., all other states are unreachable. Nevertheless, B still satis£es the properties of a FSM.)

(b) No. Consider the following counterexample:



In the arc-reversed machine, there are *two* arcs emanating from state b for the same input.

4. **10 points**

   In order for the system to be well-formed, it must have a unique non-stuttering £xed point. Using the input-output relationship

$$y(n) = cs(n) + dx(n)$$

and the feedback law $x(n) = ky(n)$, we obtain

$$y(n) = cs(n) + dk\, y(n)$$

$$y(n) = \frac{cs(n)}{1 - dk}.$$

This leads to the requirement that $1 - dk \neq 0$, or equivalently, $dk \neq 1$.