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FINAL
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Problem #1

Many network protocols require an estimate of the mean roundtrip time of a connection between the sender and receiver. (The roundtrip time is the time for a packet to get to the receiver and return to the sender.) The mean roundtrip time depends on things like the number of user currently on the network and the route of the connection.

One way of obtaining an estimate is by sending probe packets. Suppose X1, X2, ...Xn are the roundtrip times experienced by these packets; they are modeled as noisy measurements of the (unknown) mean roundtrip time mu:

Xi = mu + Zi, i = 1,..., n

where Zi are i.i.d. N(0, omega^2) random variables. Consider now two estimators of mu:

Un = (1/n)(Summation (1->n): Xi)

Vn = c(Summation (1->n): alpha^(n-i)*Xi)

where alpha is a positive constant less than 1 and c is some constant to be chosen. Vn is an exponential weighting of the past measurements.

a. An estimator mu(n) of mu is said to be unbiased if E(mu(n)) = mu. Is Un unbiased? Choose the constant c such that Vn is unbiased. (6 pts.)

b. A sequence of unbiased estimators {mu(n)} is said to be consistent if lim n->inf var (mu(n)) = 0. Are {Un} and {Vn} consistent? Give an intuitive explanation. (8 pts.)

c. In practice, the estimator Vn is often preferred over Un. Can you give a reason that is not captured by our model? (4 pts.)

Problem #2

We wish to multiplex voice calls onto a switch with outgoing link speed of C packets per time slot. At any time, each voice call has a probability p of being active and (1-p) of being silent. When it's active, it sends packets at a rate of 1 packet/time slot; when it's silent, it sends no packets. We wish to determine the number of voice calls that can be accommodated by the switch.

a. Suppose we can tolerate no packet loss from the system. What is the maximum number of calls we can accept in the system? (2pts)

b. Suppose now that we can tolerate a probability alpha of the event that more than a total of C packets arrive at the swtich at a time slot. (alpha typically small, say 10^-3). If the link speed C is high, give a good approximation of the maximum number of calls acceptable in the system. Is this number a random variable? (10 pts)

c. Call the ratio of the number in (b) to the number in (a) the statistical multiplexing gain G. This gain depends on parameters C, p, and alpha. Consider three scenarios:
i) alpha -> 1

ii) alpha -> 0
iii) C -> infinity

What does G approach in each of the 3 cases? Intuitive justification is sufficient; detailed calculations not required. For (ii), reasoning based on the approximation in (b) may not give you the right answer. Why? (6 pts)

Problem #3

The sequence X1, X2,... represents speech samples that we want to quantize using a 1-bit A/D (analog-to-digital) converter. We model each samlpe as an N(0, omega^2) random variable.

a. Consider first a strategy where we quantize each sample individually. The quantizer has the form:

q(x) = a if x>0
q(x) = -a if x<0

Find the value of a to minimize the mean-square error E[(q(Xi)- Xi)^2] and the corresponding minimum value. (Hint: Condition on an appropriate event.) Does it matter what value q(.) maps x= 0 to? (8 pts)

b. Now suppose consecutive samples are correlated with correlation coefficient rho. Instead of quantizing the Xi's, we quantize Yi delta= Xi- X'i where X'i is the MMSE estimate of Xi based on X(i-1). Using the same form of quantizer as in part (a), find the value of a that minimizes E[(q(Yi)- Yi)^2]. For what values of rho would you expect to see an improvement over the strategy in (a)? (8 pts)

Problem #4

Alice wants to deliver a packet to Bob over a network. The time to get there is random, exponentially distributed with mean mu. If Bob gets the packet within a time d, he is happy and will give Alice y dollars. If not, Bob will pay nothing.

a. Just to increase the chance that Bob will get the packet within time d, Alice decides to make n copies of the packet and send them simultaneously. If the times for each packet to get to the destination is i.i.d. with the above distribution, find the probability that at least one copy will get to Bob. (6 pts)

b. Alice is charged $1 for each copy of the packet sent as a penalty for congesting teh network. Find the optimal number of copies Alice should make to maximize her expected profit. (Note that Bob will pay only y dollars no matter how many copies he receives.) (4 pts.)

c. Suppose now that Alice has available feedback information from Bob, so that at any time she knows whether Bob has received the packet or not. So she decides to try the following strategy instead. First at time 0, she sends one copy of the packet. If by time tau < d, Bob has not received this packet copy, she will send another copy. Calculate the expected profit under this strategy. (6 pts)

d. What do you think is the optimal strategy to maximize the expected profit if d is very large? (4 pts)

Problem #5

Suppose X1, X2, ..., Xn are indepedent N(mu, omega^2) random variables.

a. Suppose we know omega^2 and we want to estimate mu from the observations X1...Xn.

i) Find the maximum likelihood (ML) estimate mu'. (4 pts)

ii) We are interested in selecting a beta such that the event mu E [mu' - beta*omega, mu + beta*omega] happens with probability 1 - alpha (alpha close to 0). Explain clearly why this is an event of a random experiment. Compute Beta. (4 pts)

b. Suppose now both mu and omega are unknown. It turns out that in this case, the ML estimate of mu remains the same as in part (a), and the ML estimate of omega is give by

omega'= sqrt(1/(n-1) * Summation(i = 1->n): (Xi-mu')^2)

i) A natural (1 - alpha) * 100% confidence interval for the parameter mu when omega is unknown is

I= [mu' - beta*omega', mu' + beta*omega']

with beta chosen such that

P(mu E I) = 1 - alpha

Let Y delta= (mu' - mu)/omega'. Compute beta in terms of the cdf of the random variable Y. Does the cdf of Y depend on the true values of the parameters mu and omega^2? Why is this important? (6 pts)

ii) Argue that mu' and Xi-mu' are jointly Gaussian. (6 pts)

iii) Show that mu' and Xi-mu' are uncorrelated. (4 pts)

iv) Using (i) and (ii) or otherwise, show that mu' and omega' are independent. (4 pts)

v) (Bonus) Consider the special case n = 2. Using part (iv) or otherwise, show that the pdf of Y delta= (mu' - mu)/ omega' is

fY(y) = sqrt(2)/(pi(1 + 2y^2)).

This allows us to compute explicitly the confidence interval in (b) (ii). (8 pts)
