EECS 126, Fall 2000
Final
Professor Chang-Hasnain

Problem #1 (15 points)

X and Y are independent Gaussian random variables with zero mean and unit variance.
$Z=X/(X+Y)$. Find the pdf of Z.

Problem #2 (10 points)

X and Y are independent Gaussian random variables with zero mean and unit variance. $Z=XY^2$. Find the pdf of Z. You do not need to carry out the integrals for this problem. Just put your answers in integral form.

Problem #3 (20 points)

The random variables X and Y are independent and each is uniform in the interval [0,a].
$Z=|X-Y|$
(a) Find the pdf of Z
(b) Find $E[|X-Y|]$

Problem #4 (30 points)

The number of customers N requesting services per hour at a given counter of the Macy's is a geometric random variable with parameter alpha. The service time per customer received is a exponential random variable with parameter beta. Let Sn be the sum of the customer service time per hour. Find:
(a) the pdf of Sn
(b) E[Sn] = ?
(c) VAR[Sn] = ?

Problem #5 (20 points)

Let X be a random variable with pdf Fx(x). Find the pdf of Y=|X| and E[Y].
(Note: The subscript of the F is an uppercase X. The x within parenthesis is lowercase.)

(a) Fx(x)=1/3   -1<=x<=2
(b) Fx(x)=e^(-x)   x>0

Problem #6 (15 points)

A transmission line card is operated with one most important component, a semiconductor diode laser, whose lifetime determines the system lifetime. System A has a single such component. System B is build with two such components: one in operation, while another one on standby and ready to kick in when the first one fails. System C is build with three such components: one in operation and two on standby. As the first one fails, the second one kicks in; and when the second one fails, the third kicks in. The lifetime of the semiconductor diode lasers used in all three systems are iid exponential random variables with parameter lambda.

(a) Find the pdf of the lifetime of the system A
(b) Find the pdf of the lifetime of the system B
(c) Find the pdf of the lifetime of the system C

Problem #7 (20 points)

The number of daily accidents is a Poisson random variable n with rate a. The probability that a single accident is fatal equals p. Find the density of, m, which is the number of fatal accidents in one day.

Problem #8 (30 points)

You are presented with the following communication system:

You are allowed to transmit binary digits. X is an element of the set {0,1}.
The channel can be modeled as additive Gaussian noise. Y=X+V. Where V~N(0,sigma^2)

a) Assume it is known to the receiver that for a given transmission the ratio of 1's to 0's is 1:9.
Find the optimal threshold (y*) in the sense that you minimize the probability of error such that your decision rule is of the form:

X = / 1 if y>y*
       \ 0 if y<y*

b) Now suppose p(X=0)=P(X=1)=1/2.

Find the optimal decision threshold (y*) such that the probability of making the mistake that a 1 was transmitted, but the receiver said it was a 0 is 0.01. Express your answer in the integral form and draw a sketch of the decision boundary and the area Perror representing the above constraint.

c) Assume that a-priori, you did not know sigma^2. The transmitter has agreed to send a long string of 0's to help the receiver decide what sigma^2 should be. The receiver knows that the mean = 0. What is the best estimate sigma^2 given the observations Y1...Yn?

Problem #9 (20 points)

Suppose X1, X2,...,X10 are independent identically distributed Poisson random variables each with a mean of 1.

a) Use Markov's inequality to get a bound on P(X1+...+X10>15)
b) Use central limit theorem to approximate P(X1+...+X10>15)

Problem #10 (20 points)

Suppose you are presented with 3 door on a game show. One of the doors has a prize behind it. You are asked to choose one door. Upon doing so, the host opens one of the doors that you did not and reveals that the prize is not behind that door (The host never chooses the door with the prize behind it). Should you change, or doesn't it matter if you change in order to win the prize? Why or why not?
