UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
Computer Sciences

Professor Tse — Fall 1998

EECS 126 — MIDTERM #1 Solutions

1a. (i) E, F independent $\Rightarrow P(E|F) = P(E) = 0.4$.

(ii) E, F mutually exclusive $\Rightarrow P(E|F) = \dfrac{P(E \cap F)}{P(F)} = 0$.

(iii) $P(E|F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{P(F)}{P(F)} = 1$.

($F \subset E$ reads, "If event F occurs, then event E must occur.)

(iv) $P(E|F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{P(E)}{P(F)} = \dfrac{0.4}{P(F)}$.

b. $P(B|A) = \dfrac{P(A|B)P(B)}{P(A)} > P(B)$ since $P(A|B) > P(A)$.

2. Let X be the lifetime of the machine picked and A_i = event that machine i is picked.

$$
\begin{aligned}
P(A_1|X \ge t) &= \frac{P(X \ge t|A_1)P(A_1)}{P(X \ge t)} \\
&= \frac{P(X \ge t|A_1)P(A_1)}{P(X \ge t|A_1)P(A_1) + P(X \ge t|A_2)P(A_2)} \\
&= \frac{[1 - F_1(t)]\,\frac{1}{2}}{[1 - F_1(t)]\,\frac{1}{2} + [1 - F_2(t)]\,\frac{1}{2}} \\
&= \frac{1 - F_1(t)}{2 - F_1(t) - F_2(t)}
\end{aligned}
$$

Note: F_1, F_2 not necessarily exponential, as some of you have assumed.

3a.

$$
\begin{aligned}
P(\text{error}) &= P(\text{output} = 1 \text{ and input} = 0) \\
&\quad + P(\text{output} = 0 \text{ and input} = 1) \\
&= P(\text{output} = 1 | \text{ input} = 0)P(\text{input} = 0) \\
&\quad + P(\text{output} = 0 | \text{ input} = 1)P(\text{input} = 1) \\
&= \varepsilon_1 p + \varepsilon_2 (1 - p)
\end{aligned}
$$

Note: Some of you wrote: $P(\text{error}) = P(\text{output} = 1 | \text{ input} = 0) + P(\text{output} = 0 | \text{ input} = 1)$

a NO-NO!

b. Let A be event that the bit gets flipped, $p(A) = \varepsilon$; let B be event that we get a tail, $p(B) = 1-\varepsilon$, where A, B are independent.

$$\begin{aligned} P(\text{error}) &= P(A)P(B) + P(A^C)P(B^C) \\ &= \varepsilon(1-\varepsilon) + (1-\varepsilon)\varepsilon \\ &= 2\varepsilon(1-\varepsilon) \end{aligned}$$

In part (a),

$$P(\text{error}) = \varepsilon < 2\varepsilon(1-\varepsilon), \text{ for } \varepsilon < \frac{1}{2}.$$

So the random rule is worse.