UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering
and Computer Sciences

EECS 126 — Fall 1995
Professor Ren

MIDTERM #1

9 October 1995

[20 pts.] 1) Prove the following statements:

a) If $p(A) = p(B) = p(A \cap B)$, then $p((A \cap B^C) \cup (B \cap A^C)) = 0$

b) If $p(A) = p(B) = 1$, then $p(A \cap B) = 1$

c) $p(A \cap B | C) = p(A | (B \cap C)) p(B | C)$

d) For any RV X, any $\alpha > 0, s > 0$, $P(X \geq \alpha) \leq e^{-s\alpha} E[e^{sX}]$

[20 pts.] 2) Box 1 contains 1000 bulbs of which 10% are defective. Box 2 contains 2000 bulbs of which 5% are defective. Two bulbs are picked from a randomly selected box.

a) Find the probability that both bulbs are defective.

b) Assuming both are defective, find the probability that they came from Box 1.

[20 pts.] 3) Random variable X has the density function

$$f_X(x) = \frac{1}{2} + \frac{1}{2}\delta\left(x - \frac{1}{4}\right), \qquad |x| \leq \frac{1}{2}$$

Find the cdf, pdf, mean, and variance of X^2.

[20 pts.] 4) The probability that a driver will have an accident in 1 month is 0.02. Find the probability that he will have 3 accidents in 100 months.

[20 pts.] 5) Players #1 and #2 roll dice alternatively starting with Player #1. The player who rolls eleven first wins. Find the probability that #1 wins.

NOTE: A Poisson RV has pmf $P_k = \frac{\alpha^k}{k!} e^{-\alpha}, \quad k = 0, 1 \ldots$