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UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
Computer Sciences

Professor Zeitouni Spring 1998

EECS 126 — MIDTERM #2

April 9, 1998, Thursday, 6-8 p.m.

☞ Please set the outline of your solution before carrying out any detailed computations.

[45 pts.] 1. Given the joint pdf of the random vector (X_1, X_2)

$$f_{X_1, X_2}(x_1, x_2) = \begin{cases} k(x_1^2 + x_2^2) & 0 \le x_1 \le 1, 0 \le x_2 \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find:

a) the value of k.

b) $F_X(x), F_Y(y), f_X(x), f_Y(y)$

c) $f_{X_1|X_2}(x_1|x_2)$. Are (X_1, X_2) independent?

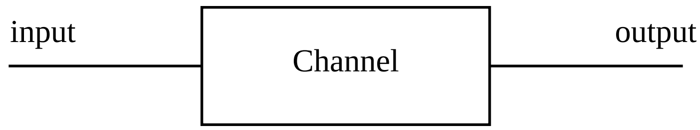
d) the MMSE estimator of X_1 given X_2. Compute the resulting mean square error.

Compute:

e) $P(|X_1 - X_2| \ge 1/3)$.

f) $P(X_1 = X_2)$.

[20 pts.] 2. A communication channel is defined as follows:



$P(\text{output bit} \neq \text{input bit}) = 0.1$.

The transmission of the string '000' means message A was transmitted; the transmission of the string '111' means message B was transmitted. Messages A and B are equally likely to be transmitted.

The receiver observes the 3 output bits corresponding to the (corrupted output) from the message transmitted.

a) Find $P(\text{output string} \neq \text{input string})$.

b) Define the decision rule:

decide message A was transmitted if in the output the majority of bits were 0, otherwise decide message B.

Compute $P(\text{error})$.

[35 pts.] 3. X_1, X_2 are independent, standard normal random variables. Let $Y = X_1 + X_2$.

Compute:

a) EX_1X_2.

b) $E(X_1|X_2)$.

c) $E(Y|X_1)$.

d) $E(X_1Y)$.

e) $E(X_1|Y)$.

f) $E(X_1Y|X_1)$.

g) $E(X_1^2|Y)$.