Name: ____________________________ Student ID No: _______________

UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
Computer Sciences

Professor Ren Fall 1997

EECS 126 — MIDTERM #2

November 17, 1997, Monday 7-9 p.m.

[42 pts.] 1. Given the joint probability density of two RVs X and Y

$$f_{XY}(x, y) = \begin{cases} k(x+y) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of k, and the cdf $F_{XY}(x, y)$. (6 pts.)

b) Find $F_X(x), F_Y(y), f_X(x), f_Y(y)$. (6 pts.)

c) Find the probability that $|X - Y| \le 1/2$. (6 pts.)

d) Find $f_{X|Y}(x|y)$. (6 pts.)

e) Find the minimum mean square error estimator of X given Y. Compute the resulting mean square error. (6 pts.)

f) Find the linear minimum mean square error estimator of X given Y. Compute the resulting mean square error. (6 pts.)

g) Are X and Y independent? Uncorrelated? Orthogonal? Explain your answer. (6 pts.)

[35 pts.] 2. An electronic system has n components. Let the lifetime of each component be X_i, $i = 1, 2, \ldots, n$, in hours. Assume that X_i, $i = 1, 2, \ldots, n$, are mutually independent, and have identical density $f_{X_i}(x) = e^{-x}, \quad x \ge 0$. Let the lifetime of the system be Y.

a) Suppose the system works only if all n components work. Find the pdf and expectation of Y. (10 pts.)

b) Suppose we already know that the system has already lasted 10 hours. Find the conditional pdf and expectation of Y. (12 pts.)

c) To increase reliability, we use redundancy by increasing the number of components from n to $2n$. Suppose the system works so long as there are at least n components working. Find the cdf of Y. (13 pts.)

[23 pts.] 3. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. RVs with mean μ and unit variance. Suppose μ is unknown.

a) Propose a scheme to estimate μ from $X_1, \ldots, X_n$. (5 pts.)

b) Suppose your estimate of μ based on $X_1, \ldots, X_n$ is denoted as $\hat{\mu}_n$. Using Central Limit Theorem, find a range of n that would guarantee the quality of the estimate in the following sense:

$$P(|\hat{\mu}_n - \mu| \leq 0.1) \geq 0.9 \text{. (18 pts.)}$$