*EECS 126 Fall 2001

Midterm 1

The exam starts at 3:40 p.m. sharp and ends at 5:00 p.m. sharp.
There are 5 problems. The maximum score is 50 points.
The exam is open book open notes.

Problem #1

For each of the following statements, indicate whether you believe that the statement is true or believe it is false, and give a briefs explanation of your reasoning. A correct answer without a valid explanation gets 1 points. A correct answer with a valid explanation gets 3 points.

(a) For any three events A, B, and C, if $P(A \cap B) > 0$ and $P(B \cap C) > 0$, then $P(A \cap C) > 0$.

(b) Given two events A and B with P(A) > 0 and P(B) > 0 , if P(A | B) > P(A) then P(B|A) > P(B).

(c) Two card are drawn at random without replacement from as standard deck of 52 playing cards
(i.e. the first card is drawn at random and then the second card is drawn at random from the remaining cards.)
Then the event that the two cards are both aces is independent of the event that they are both diamonds.

(d) g(x) is a real valued function on **R** satisfying g(x) >= x. Let X be a random variable and let Y = g(X).
The $F_Y(z) <= F_X(z)$ for all z $\in$ **R**.

(e) Adding a constant to a random variable does not change its standard deviation.

(f) A random variable X is known to satisfy $F_X(-2) = 0$ and $F_X(10) = 1$. Then it must have finite variance.

(g) If X is a random variable and $Y = X^{1/3}$, then the characteristic function of Y can be determined from the characteristic function of X.

Problem #2

7 points.

Consider the array of 25 points

{(i , j) : 1 <= i <= 5, 1 <= j <= 5}

Choose a point at random from among these. Call this point A_a. Choose another point at random from among these, independently of the choice of the first point. Call this point Q_b.

Let B denote the event that the points A_a and Q_b are either in the same row or in the same column. This includes the possibility that $Q_a = Q_b$. Another way to describe B is that it is the event where Q_a and Q_b have either the same first coordinate of the same second coordinate (or both , i.e. they are the same point).

What is the conditional distribution of Q_a among the 25 pints, conditioned on B?

Problem #3.

5 points.

There are three roads from Startville fo Endtown. A traveler chooses one of the roads at random to make the trip.

Conditioned on her choosing road i, her travel time is exponentially distributed with parameter a_i,1 <= i <= 3.

Assume time is measured in minutes.

You learn that it takes her more than 10 minutes to make the trip. What is the conditional probability that she chose road 2 to make the trip?

Problem #4.

5 points

Let X ~ Unif([-1,1]). Find Var ($X^{1/3}$).

Problem #5.

12 points

A random variable X has pdf

$$f_x(x) = \begin{cases} 1/4 & \text{if } -3 < x < -1 \\ 1/4 & \text{if } 2 < x < 4 \end{cases}$$

Let $Y = e^{|X|}$. Find f_Y.