

Name and SID:

Answer the questions on these four sheets. Show your work. Good luck.

Problem 1: (25%) You flip a fair coin repeatedly. What is the probability that you have to flip it exactly 10 times to see two “heads”?

Problem 2: (25%) Let A, B, C be three events. Assume that
 $P(A) = 0.6, P(B) = 0.6, P(C) = 0.7, P(A \cap B) = 0.3, P(A \cap C) = 0.4, P(B \cap C) = 0.4, P(A \cup B \cup C) = 1$. Find $P(A \cap B \cap C)$.

Problem 3: (25%) There are two coins. The first coin is fair. The second coin is such that $P(H) = 0.6 = 1 - P(T)$. You are given one of the two coins, with equal probabilities between the two coins. You flip the coin four times and three of the four outcomes are H . What is the probability that your coin is the fair one?

Problem 4: (25%) Define the random variable X as follows. You throw a dart uniformly in a circle with radius 5. The random variable X is equal to 2 minus the distance between the dart and the center of the circle if this distance is less than or equal to one. Otherwise, X is equal to 0.

- a. Plot carefully the probability distribution function $F(x) = P(X \leq x)$ for $x \in \mathfrak{R} := (-\infty, +\infty)$.
- b. Give the mathematical expression for the probability density function $f(x)$ of X for $x \in \mathfrak{R} := (-\infty, +\infty)$.