EECS 126 — MIDTERM #1

1. Suppose there exists a test for cancer with the following properties. Let

A = event that the test states that the tested person has cancer.

B = event that person has cancer.

$$P(B) = 0.005$$
$$P(A/B) = 0.95$$
It is known that $P(B^c/A^c) = 0.95$

What is the probability that a person has cancer given that the test says so? Is the test a good one?

2. A company produces computer chips at the defective rate of 10%. The good chips have much longer life than defective ones. The lifetime of good chips has the following cdf:

$$F_1(x) = (1 - e^{-x/10}), \qquad x \geq 0,$$

while the lifetime of defective chips has the following cdf:

$$F_2(x) = 1 - e^{-x/2}, \qquad x \geq 0.$$

Compute the pdf of the lifetime of an arbitrarily picked chip.

3. Five people want to play a game of two against two. To decide who should be left out, each of the five people tosses a fair coin.

If after one round of tossing, the result is one Head and 4 Tails, or one Tail and 4 Heads, the person whose outcome is different from the rest of the group is out. Otherwise, everyone tosses again.

What is the probability that it will take exactly *n*th round of tossing to decide?

4. RV X has the density function

$$f_x = \begin{cases} x + \frac{1}{2}\delta(x) & x \in [0, 1] \\ 0 & \text{elsewhere} \end{cases}$$

Now consider $Y = \left(X - \frac{1}{2}\right)^2$. Find its cdf, pdf, mean, variance, and standard deviation.