EECS 126 -- MIDTERM # 1 Professor Ren

October 9 , 1997, Thursday 6-8 p.m.

[20 pts.] 1. Given $P(A) = \alpha$, $P(B) = \beta$

$$P(A \cap B) = \gamma$$

Find: a) $P(A^c \cap B^c)$

b) $P(A^c \cup B^c)$

c) $P(A^c|B)$

d) $P(A^c|B^c)$

[15 pts.] 2. A committee of four is picked randomly from a pool of 5 men and 4 women. Find the probability that there will be more women than men on the committee.

[25 pts] 3. Given two coins with probability of heads being p1 for coin 1, and p2 for coin 2. You randomly pick a coin and flip it.

Let: X = the number of heads in n flippings of the randomly picked coin.

Y = the number of flippings it takes to get the first head (flipping the randomly picked coin).

a) Find the probability mass functions of X and Y, respectively.

b) Suppose you flipped k times already and still have not got a head yet. Find the probability that you picked coin 1.

[40 pts] 4. Consider a signal detector to detect if a signal is present or not, as shown below:
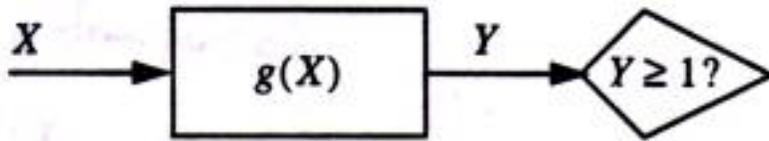
where

X is the received signal plus noise, and

$X = \{$ S, when the signal is present (with probability 1/2) $|$ M, when the signal is not present (with probability 1/2)

S is a uniform random variable in [-,2], and M is a Gaussian RV with distribution N(0,1).

a) Find the pdf of X.

b) Let $g(X) = | X - 1|$. Find the pdf of $Y = | X - 1|$.

c) Given that $Y >= 1$, find the probability that the signal is present.