DEPARTMENT OF MECHANICAL ENGINEERING
UNIVERSITY OF CALIFORNIA AT BERKELEY

Fall Semester 2006

ME104
MIDTERM # 1

1. Consider a particle P moving in a plane, and introduce polar coordinates (r, θ). Show that the velocity and acceleration of P can be written as

$$\upsilon = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta,$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\ \mathbf{e}_\theta.$$

2. A projectile P is launched from a point O with speed 130 m/s and angle of projection 40 degrees (Fig.1). It is being tracked by radar at O.

(a) Find the location of the point M at which P attains its maximum height.

(b) Calculate the values that P has for the following quantities as it passes through M: $\dot{r}, \dot{\theta}, \ddot{r}, \ddot{\theta}$.

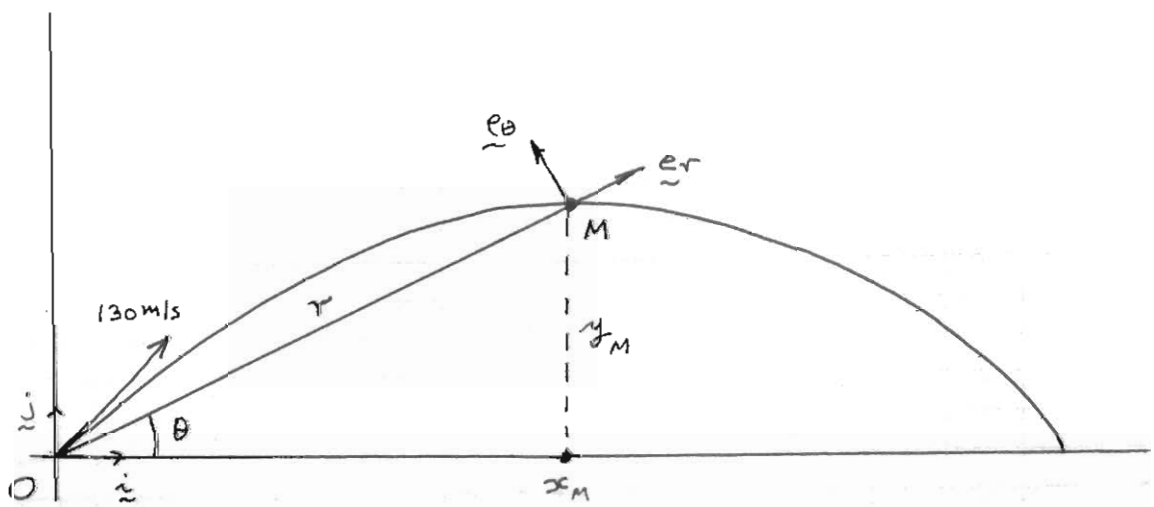
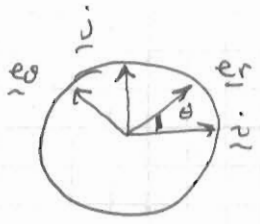


Fig. 1

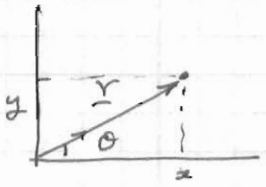
1.



$$\underset{\sim}{e}_r = \cos\theta\, \underset{\sim}{i} + \sin\theta\, \underset{\sim}{j}$$

$$\underset{\sim}{e}_\theta = -\sin\theta\, \underset{\sim}{i} + \cos\theta\, \underset{\sim}{j}$$

$$\underset{\sim}{r} = r\, \underset{\sim}{e}_r$$



$$\dot{\underset{\sim}{e}}_r = (-\sin\theta)\dot{\theta}\, \underset{\sim}{i} + (\cos\theta)\dot{\theta}\, \underset{\sim}{j} = \dot{\theta}\, \underset{\sim}{e}_\theta$$

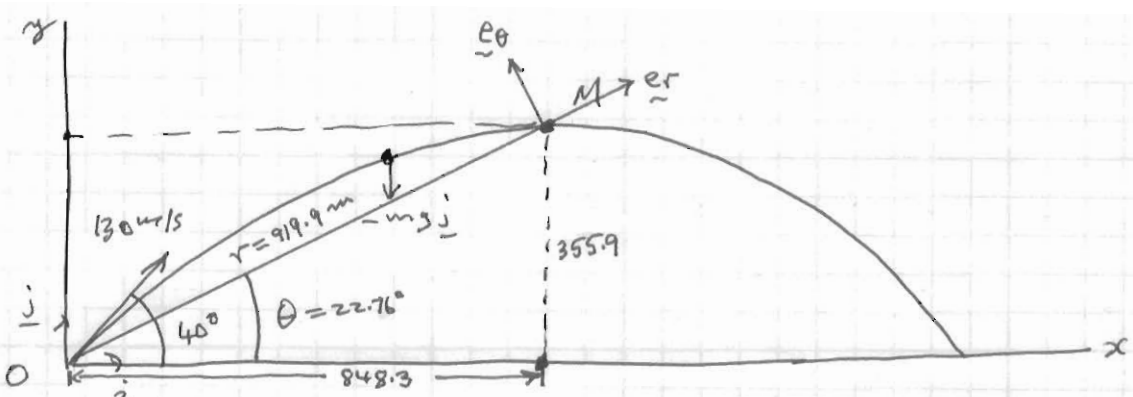
$$\dot{\underset{\sim}{e}}_\theta = (-\cos\theta)\dot{\theta}\, \underset{\sim}{i} - (\sin\theta)\dot{\theta}\, \underset{\sim}{j} = -\dot{\theta}\, \underset{\sim}{e}_r$$

$$\underset{\sim}{v} = \dot{r}\, \underset{\sim}{e}_r + r\dot{\theta}\, \underset{\sim}{e}_\theta$$

$$\underset{\sim}{a} = \ddot{r}\, \underset{\sim}{e}_r + \dot{r}(\dot{\theta}\, \underset{\sim}{e}_\theta) + \dot{r}\dot{\theta}\, \underset{\sim}{e}_\theta + r\ddot{\theta}\, \underset{\sim}{e}_\theta + r\dot{\theta}(-\dot{\theta}\, \underset{\sim}{e}_r)$$

$$= (\ddot{r} - r\dot{\theta}^2)\, \underset{\sim}{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\, \underset{\sim}{e}_\theta .$$

2.



$v_{0x} = 130\cos 40° = 99.59$ m/s, $v_{0y} = 130\sin 40° = 83.56$ m/s

$$-mg\,j = m\ddot{y}\,j + m\ddot{x}\,i$$

$$\ddot{y} = -g = \text{const.}, \quad \ddot{x} = 0$$

$$\dot{y} = -gt + v_{0y}, \quad \dot{x} = \text{const.} = v_{0x}$$

$$y = -\tfrac{1}{2}gt^2 + v_{0y}t + c_0 \qquad x = v_{0x}t + c_0$$

At M: $\dot{y} = 0$

$$gt = v_{0y} = 83.56, \quad t = 8.518\,5$$

$$x = v_{0x}(8.578) = (99.59)(8.578) = 848.3 \text{ m}$$

$$y = -\underbrace{\tfrac{1}{2}g(8.518)^2}_{355.89} + \underbrace{v_{0y}(8.578)}_{711.76} = 355.9 \text{ m}$$

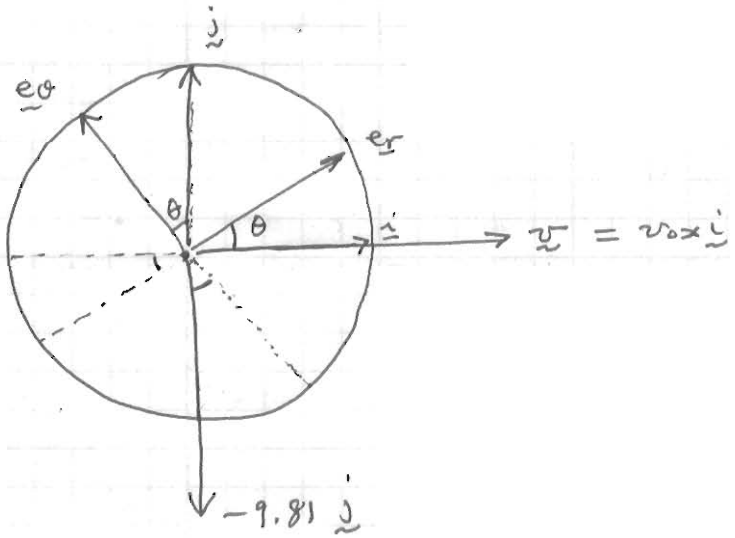
$$\theta = 22.76°$$

$$r = \sqrt{x^2+y^2} = 919.9 \text{ m}$$

At M

$$\underset{\sim}{v} = v_{0x}\,\underset{\sim}{i} = 99.59\,\underset{\sim}{i} \quad m/s$$

$$\underset{\sim}{a} = -g\,\underset{\sim}{j}$$



$$\begin{pmatrix} \underset{\sim}{e_r} \\ \underset{\sim}{e_\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \underset{\sim}{i} \\ \underset{\sim}{j} \end{pmatrix}, \qquad \begin{pmatrix} \underset{\sim}{i} \\ \underset{\sim}{j} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \underset{\sim}{e_r} \\ \underset{\sim}{e_\theta} \end{pmatrix}$$

$$\sin\theta = 0.3869 \quad , \quad \cos\theta = 0.9221$$

$$\underset{\sim}{v} = 99.59\,(\cos\theta\,\underset{\sim}{e_r} - \sin\theta\,\underset{\sim}{e_\theta}) = \underbrace{91.83}_{v_r}\,\underset{\sim}{e_r} - \underbrace{38.53}_{v_\theta}\,\underset{\sim}{e_\theta}$$

$$\underset{\sim}{a} = -9.81\,(\sin\theta\,\underset{\sim}{e_r} + \cos\theta\,\underset{\sim}{e_\theta}) = \underbrace{-3.795}_{a_r}\,\underset{\sim}{e_r} - \underbrace{9.046}_{a_\theta}\,\underset{\sim}{e_\theta}$$

$\underset{\sim}{r} = r\,\underset{\sim}{e_r}$ $\qquad$ $r = 919.9$ m

$\underset{\sim}{v} = \dot{r}\,\underset{\sim}{e_r} + r\dot{\theta}\,\underset{\sim}{e_\theta} = \underline{91.83}\,\underset{\sim}{e_r} \underline{- 38.53}\,\underset{\sim}{e_\theta}$

$\dot{r} = 91.83$ m/s

$\dot{\theta} = -\dfrac{38.53}{919.9} = -0.04188$ rad/s $= -2.400$ °/s

$\underset{\sim}{a} = (\ddot{r} - r\dot{\theta}^2)\,\underset{\sim}{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\underset{\sim}{e_\theta}$

$= -3.795\,\underset{\sim}{e_r} - 9.046\,\underset{\sim}{e_\theta}$

$\ddot{r} - r\dot{\theta}^2 = -3.795$

$\ddot{r} = -3.795 + r\dot{\theta}^2$

$= -3.795 + \underbrace{(919.9)(-0.04188)^2}_{1.6138}$

$= -2.181$ m/s^2

$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -9.046$

$r\ddot{\theta} = -9.046 - 2(91.83)(-0.04188)$

$= -9.046 + \underbrace{2(91.83)(0.04188)}_{7.6917}$

$= -1.3543$ m/s^2

$\ddot{\theta} = -0.001472$ rad/s^2

$\dot{r} = 91.8$ m/s , $\dot{\theta} = -0.0419$ rad/s,

$\ddot{r} = -2.18$ m/s^2 , $\ddot{\theta} = -0.00147$ rad/s^2

$x_M = 848$ m, $y_M = 356$ m

$r = 920$ m

$\theta = 22.8°$