MIDTERM SOLUTIONS

Problem 1: Semiconductor Fundamentals [20 points]

Consider a silicon sample maintained at 300K under equilibrium conditions, doped with the following impurities:

Phosphorus: 5×10^{16} cm^{-3} $= N_d$
Boron: 5×10^{16} cm^{-3} $= N_a$

a) What are the carrier mobilities in the sample? **[4 pts]**

$N_a + N_d = 10^{17}\ cm^{-3}$

From plot on Page 1, $\mu_n = 750\ cm^2/V\cdot s$
and $\mu_p = 300\ cm^2/V\cdot s$

$\mu_n =$ $750\ cm^2/V\cdot s$

$\mu_p =$ $300\ cm^2/V\cdot s$

b) What are the electron and hole concentrations in the sample? **[4 pts]**

$N_a - N_d = 0$ (net doping concentration is zero)

$\Rightarrow n = p = n_i$

$n =$ $10^{10}\ cm^{-3}$

$p =$ $10^{10}\ cm$ -

c) What is the conductivity type of the sample? **[2 pts]**

The sample is intrinsic.

d) What is the resistivity of the sample? **[4 pts]**

$$\rho = \frac{}{q\mu_n n + q\mu_p p} = \frac{}{q(\mu_n+\mu_p)n_i}$$

$$\frac{}{1.6\times10^{-19}(750+300)(10^{10})} = 5.95 \times 0^5\ \Omega\text{-cm}$$

$\rho \cong$ $6\times10^5\ \Omega\text{-cm}$

Problem 1 (continued)

e) What is the mean scattering time for electrons in the sample? **[3 pts]**
Assume $m_n = 0.26m_o$. Note: 1 kg-cm^2/V-s/C = 10^{-4} s

$$\mu_n = \frac{q\tau_{mn}}{m_n}$$

$$\Rightarrow \tau_{mn} = \frac{\mu_n m_n}{q} = \frac{(750)(0.26 \times 9.1 \times 10^{-31})}{1.6 \times 10^{-19}} = 1.1 \times 10^{-9} \text{ kg cm}^2/\text{V-s}/\text{C}$$

$$= 1.1 \times 10^{-13} \text{ s}$$

τ_{mn} = 0.11 ps

f) What is the hole diffusion constant in the sample? **[3 pts]**

$$D_p = \frac{kT}{q}\mu_p = (0.026)(300) = 7.8 \text{ cm}^2/\text{s}$$

D_p = 7.8 cm^2/s

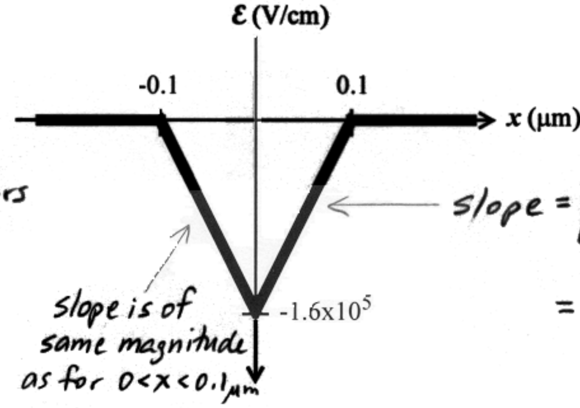
Problem 2: **pn Junction: Electrostatics [25 points]**

Given the following electric-field distribution inside an ideal Si pn step-junction maintained at 300K:

$$\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\epsilon_{Si}}$$

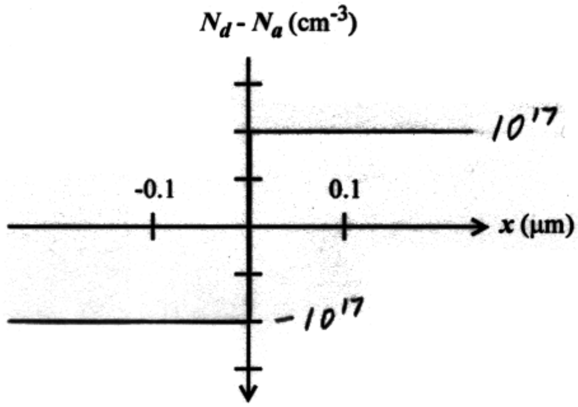
$\rho > 0 \Rightarrow$ ionized donors

$\rho < 0 \Rightarrow$ ionized acceptors



$$\text{slope} = \frac{1.6\times10^{5}}{0.1\times10^{-4}} = 1.6\times10^{10}\ \text{V/cm}^2 = \frac{qN_d}{\epsilon_{Si}}$$

a) Sketch the doping profile of this pn junction. [6 pts]



$$N_a = N_d = \frac{\epsilon_{Si} \times \text{slope}}{q} = \frac{(10^{-12})(1.6\times10^{10})}{1.6\times10^{-19}} = 10^{17}\ \text{cm}^{-3}$$

b) What is the built-in potential of this junction? [4 pts]
Note: (kT/q) ln 10 = 0.060 V at 300K

$$\phi_{bi} = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2} = \frac{kT}{q}\ln\frac{10^{34}}{10^{20}} = \frac{kT}{q}\ln 10^{14} = 14\left[\frac{kT}{q}\ln 10\right]$$

$$= 14\,(0.060) = 0.84\ \text{V}$$

ϕ_{bi} = 0.84 V

Problem 2 (continued)

c) What is the applied bias V_a for the given electric-field distribution? **[3 pts]**

$$-\text{Area under the } \mathcal{E}(x) \text{ curve} = \phi_{bi} - V_a$$

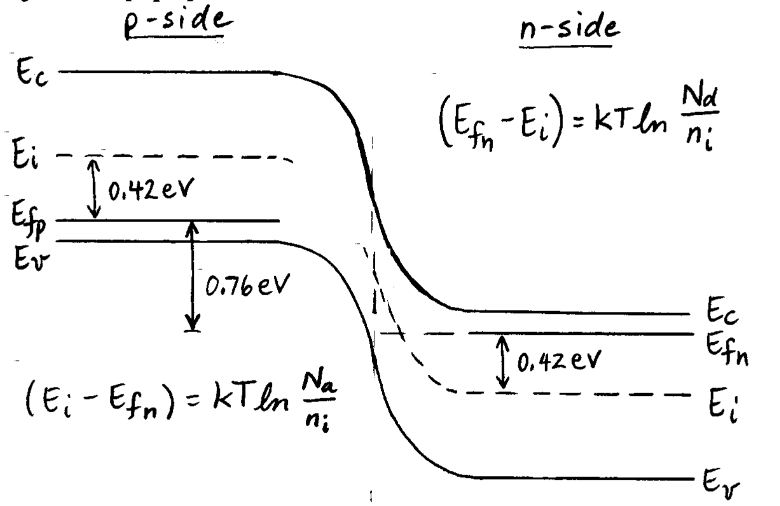
$$\text{Area} = \frac{1}{2}(0.2\times10^{-4})(-1.6\times10^{5}) = -1.6\text{ V}$$

$$1.6 = \phi_{bi} - V_a$$

$$V_a = \phi_{bi} - 1.6 = 0.84 - 1.6 = -0.76\text{ V}$$

$V_a =$ -0.76 V

d) Sketch the energy-band diagram. Show the positions of the quasi-Fermi levels relative to E_i and to each other on both sides of the junction. **[5 pts]**



$$E_{fp} - E_{fn} = -qV_a = 0.76\text{ eV}$$

e) What is the junction capacitance at this bias? **[3 pts]**

Under reverse bias, diffusion capacitance is negligible; therefore junction capacitance is comprised of only the depletion capacitance.

$$C_j = C_{dep} = \frac{\epsilon_{Si}}{W_{dep}} = \frac{10^{-12}}{0.2\times10^{-4}} = 5\times10^{-8}\text{ F/cm}^2$$

$C_j =$ 50 nF/cm^2

f) Calculate the reverse-bias breakdown voltage, assuming that the critical electric field $\mathcal{E}_{crit}$ is 5×10^{5} V/cm. **[4 pts]**

$$\mathcal{E}_p = \frac{2(\phi_{bi} - V_a)}{W_{dep}} = \sqrt{\frac{2q(\phi_{bi}-V_a)/\left(\frac{N_a N_d}{\ }\right)}{\epsilon_{Si}}} = \sqrt{\frac{q(\phi_{bi}-V_a)N}{\epsilon_{Si}}}$$

When $V_a = -V_B$, $\mathcal{E}_p = \mathcal{E}_{crit}$

$$\mathcal{E}_{crit} = \sqrt{\frac{q(\phi_{bi}+V_B)N}{\epsilon_{Si}}}$$

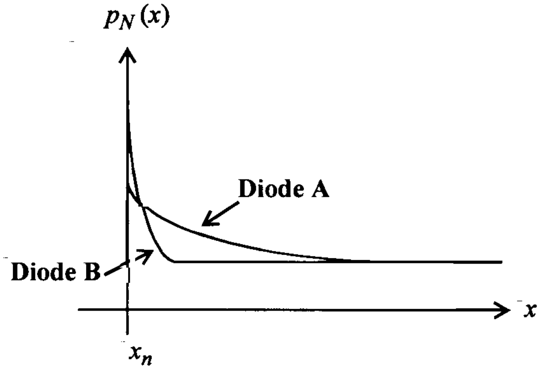
$V_B =$ 14.8 V

$$V_B = \frac{\epsilon_{Si}}{qN}\mathcal{E}_{crit}^2 - \phi_{bi}$$

$$= \frac{10^{-12}}{(1.6\times10^{-19})(10^{17})}(5\times10^{5})^2 - 0.84 = 14.8\text{ V}$$

Problem 3: pn Junctions [35 points]

a) The minority-carrier concentration profiles on the n-side of two ideal Si p^+n step-junction diodes maintained at 300K are pictured below:

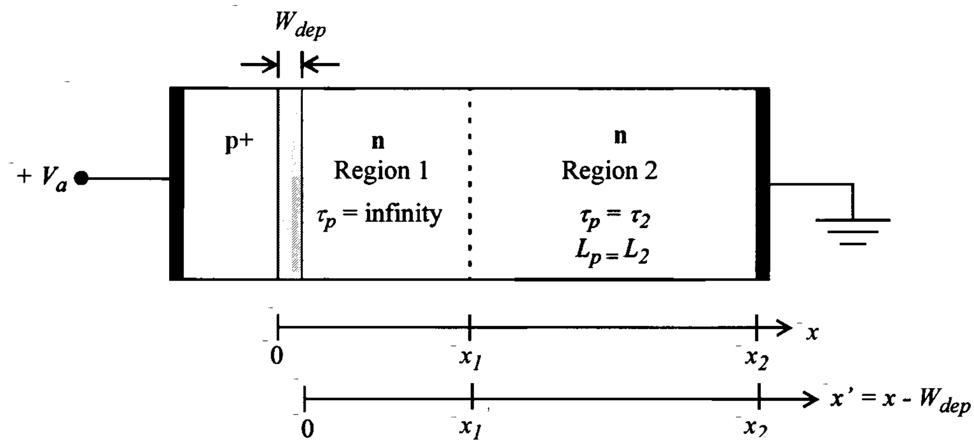


Check the appropriate boxes in the table below, and provide brief justifications for your answers. **[20 pts]**

Diode Parameter	parameter is: larger in Diode A	larger in Diode B	same for each diode	**Brief Justification**
Diode current density		✓		$\left.\frac{dp_N}{dx}\right\|_{x=x_n}$ is larger for Diode B
Doping N_d (n-side)			✓	equilibrium value of $p_N(x)$ $= \frac{n_i^2}{N_d}$ is the same for both diodes
Hole lifetime τ_p (n-side)	✓			$p_N(x)$ has a longer L_p for Diode A. $L_p = \sqrt{D_p \tau_p}$, D_p same for both diodes, since N_d is the same
Applied bias V_a		✓		$p_N(x_N) = p_{N0} e^{qV_a/kT}$ is larger for Diode B
Storage delay time t_s	✓			Stored minority carrier charge $\int_{x_n}^{\infty}(p_N(x) - p_{N0})\,dx$ is larger for Diode A

Problem 3 (continued)

b) Consider the Si p$^+$n step-junction diode below. The doping on the n-side is uniform, but the minority-carrier lifetime τ_p is infinite in Region 1 and has finite value τ_2 in Region 2. Assume that the depletion width W_{dep} is smaller than x_1, and that the length of Region 2 is much longer than the minority-carrier diffusion length L_2 in Region 2.



i) Write expressions for the excess minority-carrier concentration in Region 1 and Region 2. **[5 pts]**
Note: You should express your answers in terms of several position-dependent parameters.

Region 1: minority-carrier diffusion equation: $\frac{\partial p_N'}{\partial t} = D_p \frac{\partial^2 p_N'}{\partial x'^2} - \frac{p_N'}{\tau_p} = D_p \frac{\partial^2 p_N'}{\partial x'^2} = 0$ ($\tau_p \to \infty$)

General solution is $p_N'(x') = C_1 x' + C_2$

Region 2: minority-carrier diffusion equation: $\frac{\partial p_N'}{\partial t} = D_p \frac{\partial^2 p_N'}{\partial x'^2} - \frac{p_N'}{\tau_2} = 0$

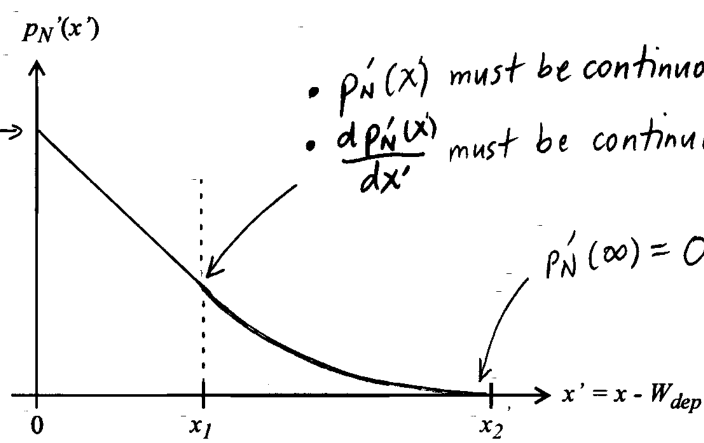
General solution is $p_N'(x') = C_3 e^{-x'/L_2} + C_4 e^{x'/L_2}$ where $L_2 = \sqrt{D_p \tau_2}$

Region 1: $p_N'(x') =$ $C_1 x' + C_2$

Region 2: $p_N'(x') =$ $C_3 e^{-x'/L_2} + C_4 e^{-x'/L_2}$

ii) Sketch the excess minority-carrier profile in the quasi-neutral n region under forward bias. Specify the boundary conditions at $x' = 0$, x_1' and x_2'. **[10 pts]**

$p_N'(0) = \frac{n_i^2}{N_d}(e^{qV_a/kT} - 1) \rightarrow$



- $p_N'(x)$ must be continuous at $x' = x_1'$
- $\frac{dp_N'(x)}{dx'}$ must be continuous at $x' = x_1'$

$p_N'(\infty) = 0$

4 boundary conditions → can solve for C_1, C_2, C_3 and C_4
(These will be dependent on V_a.)

Problem 4: Metal-Semiconductor Contact [20 points]

Consider an **ideal** Schottky diode maintained at 300K, made by depositing tungsten ($q\Psi_M = 4.5$ eV) onto n-type Si.

a) What is the work function of Si, if $N_d = 10^{17}$ cm^{-3}? **[5 pts]**

$$E_f - E_i = kT \ln \frac{N_d}{n_i} = kT \ln \frac{10^{17}}{10^{10}} = 7(kT \ln 10) = 7(0.060) = 0.42 \text{ eV}$$

$$E_c - E_f = \frac{1}{2} E_g - (E_f - E_i) = 0.56 - 0.42 = 0.14 \text{ eV}$$

E_f ——— E_c

- - - - - - - E_i

——— E_v

$$\Psi_{Si} = \chi_{Si} + \frac{E_c - E_f}{q} = 4.05 + 0.14 = 4.19 \text{ V}$$

$\Psi_{Si} =$ 4.19 V

b) What is the Schottky barrier height? **[3 pts]**

$$\phi_{Bn} = \Psi_M - \chi_{Si} = 4.5 - 4.05 = 0.45 \text{ V}$$

$\phi_{Bn} =$ 0.45 V

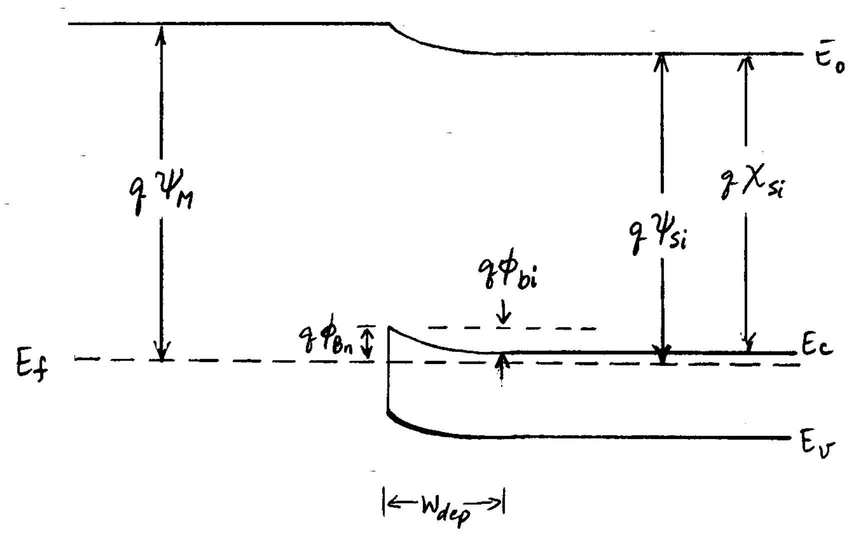
c) What is the built-in potential? **[3 pts]**

$$\phi_{bi} = \phi_{Bn} - \frac{E_c - E_f}{q} = 0.45 - 0.14 = 0.31 \text{ V}$$

$\phi_{bi} =$ 0.31 V

Problem 4 (continued)

d) Draw the equilibrium energy-band diagram of the Schottky diode. Label $q\Psi_M$, $q\Psi_{Si}$, $q\chi_{Si}$, $q\phi_{Bn}$, and $q\phi_{bi}$, as well as E_c, E_v, E_i and E_f in the Si. **[9 pts]**



$$W_{dep} = \sqrt{\frac{2\epsilon_{Si}\phi_{bi}}{qN_d}} = \sqrt{\frac{2(10^{-12})(0.31)}{(1.6\times10^{-19})(10^{17})}} = 6.2\times10^{-6}\ \text{cm}$$

$$= 620\ \text{Å}$$