University of California, Berkeley *Fall 1996*
EE130 Integrated Circuit Devices *Prof. J. Bokor*

MIDTERM EXAM 1

NAME: Solutions
SIGNATURE:
STUDENT ID #:

CLOSED BOOK. ONE 8 1/2" X 11" SHEET OF NOTES, AND SCIENTIFIC POCKET CALCULATOR PERMITTED.

TIME ALLOTTED: 80 MINUTES.

TREAT THIS EXAM AS A BOOKLET. DO NOT REMOVE STAPLES.

ALL ANSWERS MUST BE WRITTEN IN THE INDICATED BOXES.

SHOW ALL WORK AS CLEARLY AS POSSIBLE TO MAXIMIZE OPPORTUNITY FOR PARTIAL CREDIT.

UNLESS SPECIFIED OTHERWISE, NUMERICAL VALUES MUST BE CALCULATED TO AN ACCURACY OF 2 SIGNIFICANT FIGURES OR BETTER.

IF YOU USE AN EXTRA SHEET OF SCRATCH PAPER, WRITE YOUR NAME ON IT AND INSERT IT INTO THE EXAM BOOKLET.

Numbers you might need:

For ease of grading, please use these values in your calculations in preference to any others you may have on your sheet.

Boltzmann's constant, $k = 1.38 \times 10^{-23}\ J/K$

Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-14}\ F/cm$

Electron charge, $q = 1.6 \times 10^{-19}\ C$

Free electron mass, $m_0 = 9.1 \times 10^{-31}\ kg$

Thermal voltage, $kT/q = 0.0258\ V$ (at 300K)

Relative dielectric constant of silicon, $K_s = 11.8$

Effective masses in silicon at 300K. Electrons: $m_n^* = 1.18\ m_0$; Holes: $m_p^* = 0.81\ m_0$

Silicon band gap at 300K, $E_g = 1.12\ eV$

Intrinsic carrier density in silicon at 300K, $n_i = 10^{10}\ cm^{-3}$

Table 1: Mobilities in silicon $(cm^2\ V^{-1}\ s^{-1})$

N (cm^{-3})	Arsenic	Phosphorous	Boron
10^{13}	1423	1424	486
10^{14}	1413	1416	485
10^{15}	1367	1374	478
10^{16}	1184	1194	444
10^{17}	731	727	328
10^{18}	285	279	157
10^{19}	108	115	72

Problem 1. Consider two silicon samples. Sample 1 is phosphorous doped n-type with donor concentration $N_D = 10^{17}\ cm^{-3}$; Sample 2 is boron doped p-type with acceptor concentration $N_A = 10^{16} cm^{-3}$.

a) **[6 points]** Find the resistivity (in units of Ω-cm) of each sample at 300K.

$\rho = \frac{1}{nq\mu_n}$ n-type; $\rho = \frac{1}{nq\mu_p}$ p-type

Sample 1: (n-type)
$n = 10^{17}\ cm^{-3}$ $\mu_n = 727\ cm^2/V\text{-}S$ (table 1)
$\rho = 0.0086\ \Omega\text{-}cm$

Sample 2: (p-type)
$p = 10^{16}\ cm^{-3}$ $\mu_p = 444\ cm^2/V\text{-}S$
$\rho = 1.408\ \Omega\text{-}cm$

Resistivity of Sample 1:	$0.086\ \Omega\text{-}cm$	**(3 points)**
Resistivity of Sample 2:	$1.408\ \Omega\text{-}cm$	**(3 points)**

b) **[7 points]** Find the position of the Fermi level referred to the valence band edge, E_v, or the conduction band edge, E_c, in each material at 300K to within an accuracy of 1 meV. Repeat for Sample 3, which has both types of impurities present in the same respective amounts, i.e. $10^{17}\ cm^{-3}$ of phosphorous and $10^{16}\ cm^{-3}$ of boron.

Sample 1 and Sample 3 are n-type:

$$n = \frac{N_D - N_A}{2} + [(\frac{N_D - N_A}{2})^2 + n_i^2]^{\frac{1}{2}} = 10^{17}\ cm^{-3} \text{ Sample 1}$$
$$= 9 \times 10^{16}\ cm^{-3} \text{ Sample 3}$$

$$E_F - E_i = kT \ln(\frac{n}{n_i}) = 0.416\ eV \text{ Sample 1}$$
$$= 0.413\ eV \text{ Sample 3}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln(\frac{m_p^*}{m_n^*}) = 0.553\ eV \text{ (relative to } E_v = 0)$$

$$\text{so } E_F = E_i + 0.416\ eV = \boxed{\begin{array}{l} 0.969\ eV \text{ above } E_v \\ 0.151\ eV \text{ below } E_c \end{array}} \text{ Sample 1}$$

$$E_F = E_i + 0.413\ eV = \boxed{\begin{array}{l} 0.966\ eV \text{ above } E_v \\ 0.154\ eV \text{ below } E_c \end{array}} \text{ Sample 3}$$

Sample 2 is p-type $\quad p = 10^{16}\ cm^{-3}$

$$E_i - E_F = kT \ln(\frac{p}{n_i}) = 0.356\ eV$$

$$E_F = E_i - 0.356\ eV = \boxed{0.197\ eV \text{ above } E_v} \text{ Sample 2}$$

Fermi level in Sample 1: -0.151 eV relative to [$\boxed{E_c}$, E_v]	(choose and circle one)	**(2 points)**
Fermi level in Sample 2: +0.197 eV relative to [E_c, $\boxed{E_v}$]	(choose and circle one)	**(2 points)**
Fermi level in Sample 3: -0.154 eV relative to [$\boxed{E_c}$, E_v]	(choose and circle one)	**(3 points)**

c) **[7 points]** Find the equilibrium minority carrier densities at 300K in each of the samples.

$n_0 p_0 = n_i^2$

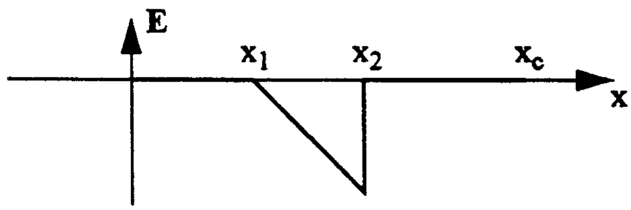
<u>Sample 1</u> $\quad n_0 = 10^{17}\ cm^{-3} \quad \boxed{p_0 = 10^3\ cm^{-3}}$

<u>Sample 2</u> $\quad p_0 = 10^{16}\ cm^{-3} \quad \boxed{n_0 = 10^4\ cm^{-3}}$

<u>Sample 3</u> $\quad n_0 = 9 \times 10^{16}\ cm^{-3} \quad \boxed{p_0 = 1.11 \times 10^3\ cm^{-3}}$

Minority carrier density in Sample 1: 10^3 cm^{-3} Minority carrier type [$\boxed{\text{holes}}$, electrons] (circle one) **(2 points)**
Minority carrier density in Sample 2: 10^4 cm^{-3} Minority carrier type [holes, $\boxed{\text{electrons}}$] (circle one) **(2 points)**
Minority carrier density in Sample 3: 1.11×10^3 cm^{-3} Minority carrier type [$\boxed{\text{holes}}$, electrons] (circle one) **(2 points)**

Problem 2. The equilibrium electric field distribution inside of a Si device is somehow maintained as pictured below, with $N_A = 10^{17}\ cm^{-3}$, for $0 \le x \le x_1$, and $N_D = 10^{17}\ cm^{-3}$, for $x_2 \le x \le x_c$.

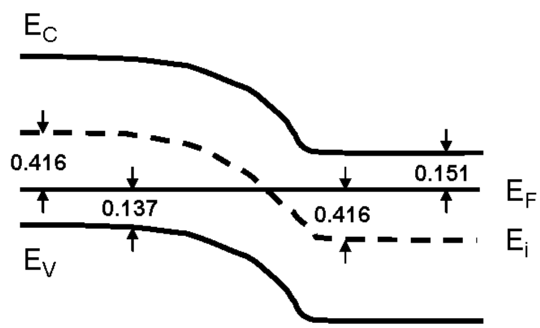


a) **[10 points]** Draw the energy band diagram for this device. Include E_C, E_V, E_i, and E_F on your diagram, and be quantitative (on both sides, indicate energy differences between E_F, E_i, and at least one of the band edges).

from 1(b): $E_F = E_i = 0.416$ n side
$E_i - E_F = 0.416$ p side

for linear E-field, potential is quadratic

(energies in eV)

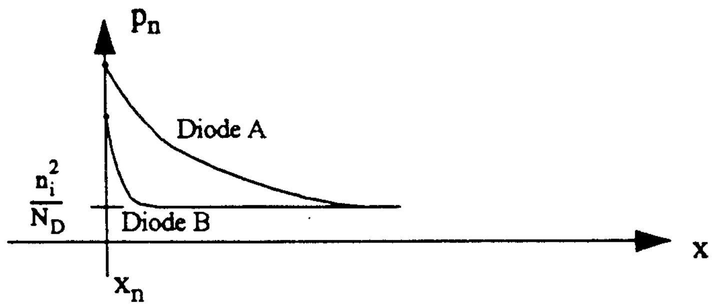


b) **[10 points]** What is the electrostatic potential drop across the device, $V(x = x_c) - V(x = 0)$?

$V_{b_i} = 2 \cdot (0.416) = +0.832$

Voltage drop across device: $+0.832$ V. (Be sure to indicate sign)

Problem 3. A linear-scale plot of the minority carrier concentration on the n-side of two ideal p^+-n diodes maintained at room temperature is pictured below. The n-side doping, N_D, and the area, A are the same in both diodes. Answer the following questions: (circle the correct choice).



a) Diode A is

[$\boxed{\text{(i) forward biased}}$, (ii) zero biased, (iii) reverse biased]?

b) Diode B is

[$\boxed{\text{(i) forward biased}}$, (ii) zero biased, (iii) reverse biased]?

c) The magnitude of the bias applied to Diode B is

[(i) larger than, (ii) equal to, $\boxed{\text{(iii) less than}}$]

the magnitude of the bias applied to Diode A?

d) The magnitude of the DC current, flowing through Diode B is

[$\boxed{\text{(i) larger than}}$, (ii) equal to, (iii) less than]

the magnitude of the DC current, flowing through Diode A?

e) The minority carrier lifetime, τ_p, in Diode B is

[(i) longer than, (ii) equal to, $\boxed{\text{(iii) shorter than}}$]

the minority carrier lifetime in Diode A?

Problem 4.

a) **[10 points]** An n^+p step junction diode maintained at 300K has a p-side doping of $N_A = 10^{17}\ cm^{-3}$, and a p-side thickness of $x_c = 1\ \mu m$. Determine the punch-through voltage. [Hint: assume the Fermi-level is positioned at the band-edge inside the n^+ region.]

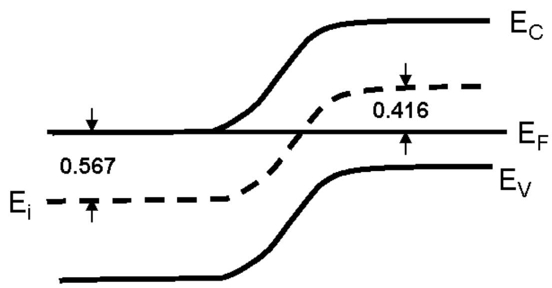
$$x_p = [\frac{2\cdot(V_{b_i} - V_A)K_s\epsilon_0}{qN_A}]^{\frac{1}{2}}$$

<u>Solve for V_A</u>

$$V_A = V_{b_i} - \frac{qN_A x_p^2}{2K_s\epsilon_0}$$

<u>find V_{b_i}</u>

band diagram



$$V_{b_i} = 0.567 + 0.416\ V$$
$$= 0.983\ V$$

$$V_A = 0.983 - 76.6 = \boxed{-75.62}$$

Punch-through voltage: -75.62 V. (Be sure to indicate sign)

b) **[10 points]** Determine the junction capacitance per unit area (F/cm^2) of this diode at zero bias and $20\ V$ reverse bias.

$C_J = \frac{K_s \epsilon_0 A}{W}$

$W = [\frac{2 \cdot (V_{b_i} - V_A) K_s \epsilon_0}{q N_A}]^{\frac{1}{2}}$ $\qquad V_{b_i} = 0.983$

$= 1.14 \times 10^{-5} \cdot (V_{b_i} - V_A)^{\frac{1}{2}}\ cm$

$= 1.13 \times 10^{-5}\ cm$ at $V_A = 0\ V$

$= 5.27 \times 10^{-5}\ cm$ at $V_A = -20\ V$

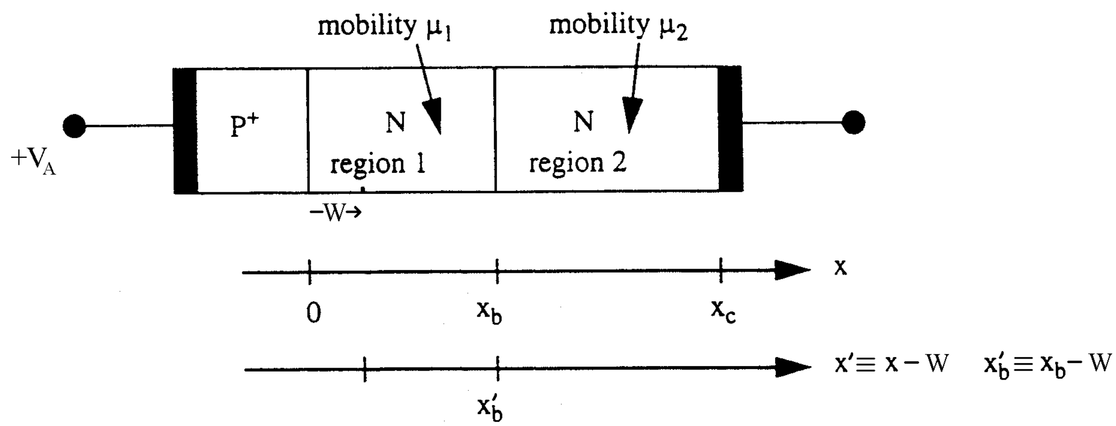
$C_J = \frac{1.04 \times 10^{-12}}{W}\ F/cm^2$

$= 9.24 \times 10^{-8}\ F/cm^2$ at $V_A = 0\ V$

$= 1.98 \times 10^{-8}\ F/cm^2$ at $V_A = -20\ V$

Junction capacitance at zero bias: 9.28×10^{-8} F/cm^2.
Junction capacitance at $20\ V$ reverse bias: 1.98×10^{-8} F/cm^2.

Problem 5. **[25 points]** Consider the silicon p^+-n step junction diode with cross-sectional area A, below. The doping and minority carrier lifetime, τ_p, are uniform throughout the n side, but the hole mobility, μ_p, has the value μ_1, in region 1 ($0 \le x \le x_b$), and μ_2, in region 2 ($x_b \le x \le x_c$). Assume that the minority carrier diffusion length is much shorter than the length of region 2, i.e. $L_p \ll x_c - x_b$ in the region $x_b \le x \le x_c$. Further assuming that the depletion width, W, never exceeds x_b for all biases of interest, and exluding biases that would cause high level injection, breakdown, or significant series resistance effects, derive an expression for the room temperature I-V characteristic of the diode. **[You may express your answer in terms of seeral voltage dependant parameters, which are to be determined by solving a set of simple linear equations. If you clearly show the equations to be solved and the parameters to be solved for, you need not actually carry out the solution.]**



region 1 $\quad \Delta p(x') = A_1 e^{-x'/L_1} + A_2 e^{x'/L_1} \qquad L_1 = \sqrt{D_1 \tau};\ D_1 = \frac{kT\mu_1}{q}$

region 2 $\quad \Delta p(x') = A_3 e^{-x'/L_2} + A_4 e^{x'/L_2} \qquad L_2 = \sqrt{D_2 \tau};\ D_2 = \frac{kT\mu_2}{q}$

Boundary conditions:

(1) $\Delta p(x=0) \equiv \Delta p_0 = \frac{n_i^2}{N_D} \cdot (e^{qV_A/kT} - 1)$
$\Longrightarrow A_1 + A_2 = \Delta p_0$

(2) region 2 is "long"
$\Delta p(x \to \infty) \to 0$
$\Longrightarrow A_4 = 0$

(3) carrier density is continuous
$\Delta p(x = x_b^-) = \Delta p(x = x_b^+)$
$\Longrightarrow A_1 e^{-x_b'/L_1} + A_2 e^{x_b'/L_1} = A_3 e^{-x_b'/L_2}$

(4) current is continuous
$D_1 \frac{\partial \Delta p}{\partial x} \mid_{x'=x_b'^-} = D_2 \frac{\partial \Delta p}{\partial x} \mid_{x'=x_b'^+}$
$\Longrightarrow -A_1 \frac{D_1}{L_1} e^{-x_b'/L_1} + A_2 \frac{D_1}{L_1} e^{x_b'/L_1} = A_3 \frac{D_2}{L_2} e^{-x_b'/L_2}$

<u>current</u>
$J = J_p(x=0) = -qD_1 \frac{\partial \Delta p}{\partial x} \mid_{x'=0} = -\frac{qD_1}{L_1} \cdot (A_1 - A_2)$

Linear equations to be solved for unknown parameters:

(1) $A_1 + A_2 = \Delta p_0$

(2) $A_1 e^{-x_b'/L_1} + A_2 e^{x_b'/L_1} = A_3 e^{-x_b'/L_2}$

(3) $-A_1 \frac{D_1}{L_1} e^{-x_b'/L_1} + A_2 \frac{D_1}{L_1} e^{x_b'/L_1} = A_3 \frac{D_2}{L_2} e^{-x_b'/L_2}$

Solving these gives voltage dependent $A_1(V_A)$, $A_2(V_A)$, $A_3(V_A)$

Current-voltage relation:

$I = AJ = -\frac{AqD_1}{L_1} \cdot (A_1 - A_2)$