UNIVERSITY OF CALIFORNIA, BERKELEY
FALL SEMESTER 2005

FINAL EXAMINATION

(CE130-1 Mechanics of Materials)

Problem 1: (10 points)

A pin-jointed 3-column structure is shown in the Figure 1. There is an external force, F, acting on the point C. When external force F is below a critical load, F_{cr}, the position shown in the figure is the initial equilibrium position. (1) Find internal axial forces for column AC, BC, and CD; (2) Find the critcal load F_{cr} ?

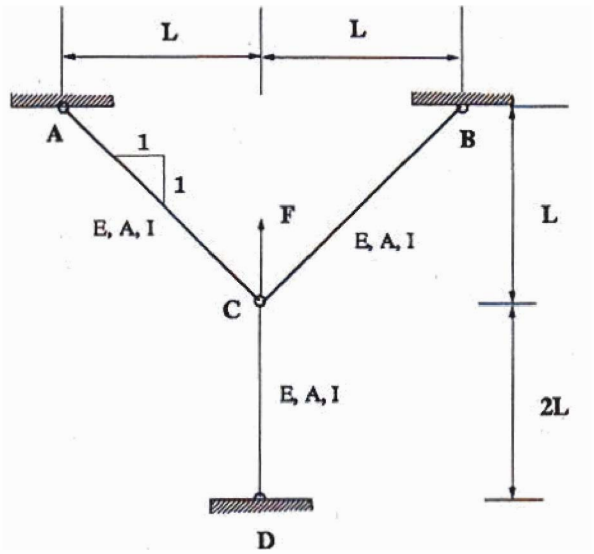

Figure 1: Schematic illustration of problem 1

(Hint: (1) use Castigliano's second theorèm, the energy for axially deformed column is, $U = \dfrac{P^2L}{2EA}$, where P is the axial force, L is the length of the column, E is the Young's modulus, and A is the cross section of the column; (2) Euler's formula $P_{cr} = \pi^2 EI/L^2$.)

Problem 2 (10 points)

A simply supported beam subjected two concentrated forces that have the same magnitude, P, shown in Figure 2. Draw shear and moment diagrams. (10 points)

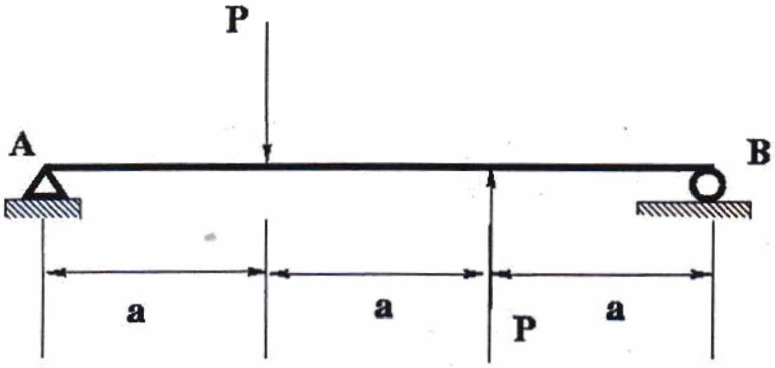

Figure 2: Simply supported beam with concentrated forces.

Problem 3: (15 points)

Consider a plane stress state at one material point as follows

$$\sigma = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix} \quad \text{(MPa)}$$

A. draw Mohr's circle of the stress state at that point;

B. find principal stresses σ_1, σ_2, and show the results on properly oriented element in physical space;

C. find the maximum shear stress, and show the results on properly oriented element in physical space;

D. find at least one angle between the right face of the initial infinitesimal element and the planes on which the normal stress is zero.

(Hint:

$$\begin{aligned}
\sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
\sigma_2 &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
tan2\theta_p &= \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} \\
tan2\theta_s &= -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}
\end{aligned}$$

)

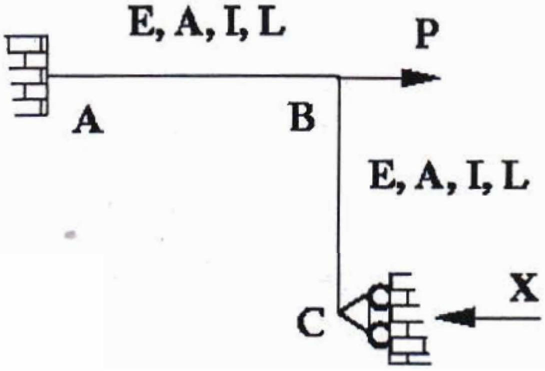

Figure 3: Problem 4

Problem 4 (15 points)

For the planar frame shown below, apply Castigliano's 2nd theorem to determine the vertical deflection at point B. Take into account the strain energy contribution due to bending deformation and axial deformation. Young's modulus, E, the cross section, A, the moment of inertia, I, and the length of each segment of frame, L, are all constant.

Hints:

$$\begin{aligned} U_{bend} &= \frac{1}{2EI}\int_0^L M(s)^2 ds \\ U_{axial} &= \frac{1}{2EA}\int_0^L P(s)^2 ds \end{aligned} \quad (1)$$

Problem 5 (10 points)

Consider an infinitesimal element shown in Figure 4. Use Mohr's circle method to find: the normal stresses and shear stresses on the vertical plane. The traction components on the two oblique planes are given in Figure 4.

Problem 6 (15 points)

A box beam is made by nailing together four boards in the configurations shown in Figure 5 and labeled as *Config.1* and *Config.2*. The beam supports a concentrated load of 1000 N at its midspan, and it rests on simple supports as shown in Figure 5. Assume each nail can withstand an allowable shear force of 200 N.

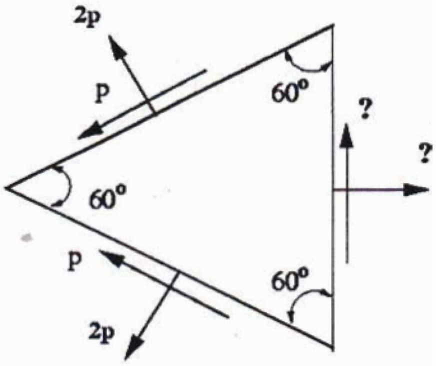

Figure 4: Problem 5

a . draw shear diagram;

b . what is the maximum spacing (Δ_s) for configuration 1;

c . what is the maximum spacing (Δ_s) for configuration 2;

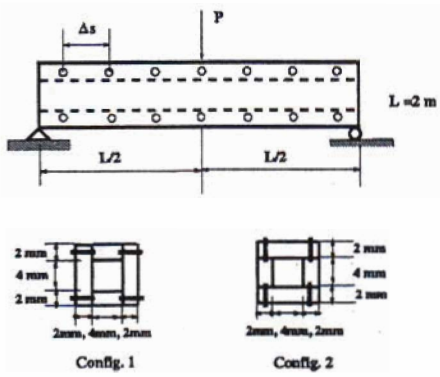

Figure 5: Problem 6

(Hint:

$$
\begin{aligned}
q &= \frac{VQ}{I_z}; \\
Q &= \int_A y dA = \bar{y}A \\
q &= \frac{N_{allowable}}{\Delta_s}
\end{aligned}
$$

where $N_{allowable}$ stands for allowable shear force by the nails.)

Problem 7 (10 points)

A planar frame ABCD is subjected a concentrated moment, M, at the point C as shown in Figure 6.

(a) draw the real moment diagram along the frame;

(b) find the vertical displacement at point D, i.e. v_V^C ?

(Hint:

(1) use virtual force method,

$$\bar{f} \times v_V^D = \int \bar{m}_c(s) \frac{M(s)}{EI} ds$$

where s is a local coordinate;

or (2) use Castigliano's second theorem,

$$v_V^D = \frac{\partial U^*}{\partial P'} = \frac{\partial U}{\partial P'}$$

Only taking into account strain energy due to bending mode:

$$U = \frac{1}{2EI} \int_0^L M(s)^2 ds$$

)

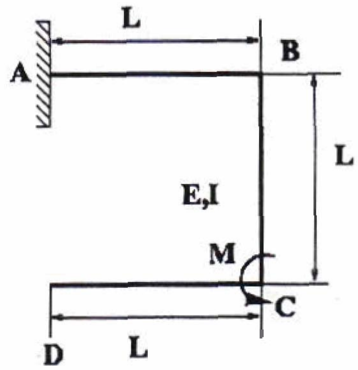

Figure 6: Problem 7

Problem 8 (15 points)

A L-shaped beam is made of a rectangular section and a solid cylinder section with radius $R = 0.1m$. The span of the both section is $L = 2.0\ m$. There is a concentrated horizontal force, $F = 300N$, acting on the free-end of the

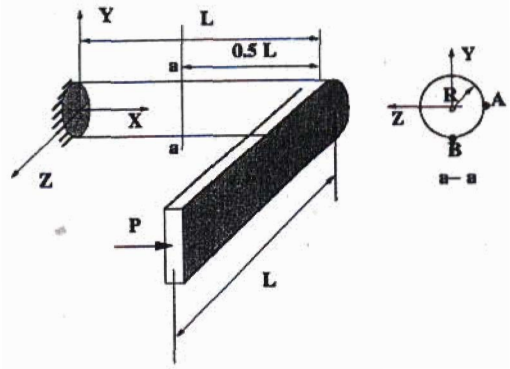

Figure 7: Problem 8

rectangular section (as shown in Figure 7.). (1) Draw the moment diagram, shear diagram, and internal torque diagram; (2) Find the normal stress σ_x, shear stresses τ_{xy} and τ_{xz} at point A; (3) Find the normal stress σ_{xx} and shear stress τ_{xy} and τ_{xz} at point B;

Hints:

$$
\begin{aligned}
\sigma_x &= -\frac{M_z y}{I_z} \\
\tau &= \frac{VQ(y)}{I_z t} \\
\tau &= \frac{T\rho}{I_p} \\
I_z &= \frac{1}{2} I_p = \frac{R^4\pi}{4} \\
\text{For semi-circle,} & \\
Q(y)\Big|_{y=0} &= \frac{2}{3}R^3
\end{aligned}
$$