Department of Physics
University of California, Berkeley

Mid-term Examination 1
Physics 7B, Section 2

6:00 pm - 8:00 pm, 24 February 2004

Name:____

SID No:_______________________________________

Discussion Section:____________________________

Name of TA:___________________________________

Score:____

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	

Answer all six problems. Write clearly and explain your work. Partial credit will be given for incomplete solutions provided your logic is reasonable and clear. Cross out any parts that you don't want to be graded. Enclose your answers with boxes. **Express all numerical answers in SI units**. Answers with no explanation or disconnected comments will not be credited. If you obtain an answer that is questionable, explain why you think it is wrong.

Constants and Conversion factors

Avogadro number, N_A	6.022×10^{23}
Universal gas constant, R	$8.315\ \mathrm{J \cdot mol^{-1} \cdot K^{-1}} = 1.99\ \mathrm{cal \cdot mol^{-1} \cdot K^{-1}}$
Boltzmann constant, k	$1.381 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$
Stefan-Boltzmann constant, σ	$5.67 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$
Acceleration due to gravity, g	$9.8\ \mathrm{m \cdot s^{-2}}$
Specific heat of water	$1\ \mathrm{kcal \cdot kg^{-1} \cdot {}^{\circ}C^{-1}}$
Heat of fusion of water	$80\ \mathrm{kcal \cdot kg^{-1}}$
1 atm	$1.013 \times 10^5\ \mathrm{N \cdot m^{-2}}$
1 kcal	4.18×10^3 J
1 hp	746 W 1 liter $10^3\ \mathrm{cm^3}$

Some useful equations

$$
\begin{aligned}
\Delta L &= \alpha L \Delta T \\
W &= \int p dV \\
Q &= mc\Delta T \\
Q &= nC\Delta T, \text{ where } C = C_V \text{ or } C_P \\
\overline{v} &= \sqrt{\frac{8kT}{\pi m}} \\
\frac{dN_v}{dv} &= 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} \\
PV^\gamma &= constant \quad TV^{\gamma-1} = constant \quad \gamma = \frac{C_P}{C_V} \\
\frac{dQ}{dt} &= \sigma\epsilon A T^4 \\
\frac{dQ}{dt} &= -\kappa A \frac{dT}{dx}
\end{aligned}
$$

1. [25 points] Short Questions

(a) [5 points] Circle T or F for True or False

T F (i) The heat capacity of a body is the amount of heat that it can store at a given temperature.

T F (ii) If the absolute temperature of a body triples, the rate at which it radiates energy increases by a factor of 9.

T F (iii) When a system goes from equilibrium state 1 to state 2, the change in the internal energy is the same for all processes.

T F (iv) The internal energy of a given amount of an ideal gas at equilibrium depends only on its absolute temperature.

T F (v) For any material that expands when heated, C_P is greater than C_V.

T F (vi) Pressure and Temperature are state variables.

T F (vii) For a solid the mean distance between atoms is 2×10^{-10} m.

T F (viii) The Carnot efficiency changes by the same amount from either increasing the hotter temperature reservoir by 10% or decreasing the lower reservoir temperature by 10% to within 1%.

T F (ix) A good emitter of radiation is a good absorber of radiation.

T F (x) According to the principle of equipartition, energy is shared equally among the active degrees of freedom in an amount of $\frac{1}{2}kT$ on average.

(b) [5 points] Circle correct answer

(i) A scuba diver releases a spherical air bubble 3-cm diameter at a depth of 14-m in water at 298 K. Approximately what is the air bubble's radius just as it reaches the surface, if the temperature stays constant. Take the density of water to be 1000 kg/m^3.
(A) 3 cm.
(B) 4 cm.
(C) 5 cm..
(D) 6 cm
(E) 7 cm.

(ii) You are on a trip overseas and it seems very cold. You use a local thermometer to see how cold it really is It reads -40°C, what temperature is that in ° F? (A) -104° F.
(B) -60° F.
(C) -40° F.
(D) -20° F.
(E) -10° F..

(iii) A 1 meter radius balloon containing an ideal gas is cooled from 400 K to 300 K. How much does its volume and surface area change assuming the external pressure is constant?
(A) decrease by 33% and 21%.
(B) decrease by 25% and 17%.
(C) no change.
(D) increase by by 25% and 17%.
(E) increase by 33% and 21%.

(iv) A 1 meter radius glass sphere is heated by 100°C. About how much does its surface area increase, given $\alpha = 9 \times 10^{-6}/°\mathrm{C}$?
(A) increase by 3 cm^2.
(B) increase by 50 cm^2.
(C) increase by 100 cm^2.
(D) increase by 200 cm^2
(E) increase by 300 cm^2

(v) At standard temperature (273 K) and pressure (1 atm) how many air molecules are there per cubic cm?
(A) 269 molecules/cm^3.
(B) 2.69×10^{19} molecules/cm^3.
(C) 2.69×10^{23} molecules/cm^3.
(D) 6.02×10^{23} molecules/cm^3.
(E) 2.69×10^{25} molecules/cm^3.

(c) [5 points] Circle correct answer

(i) Two gases are mixed and in equilibrium, the ratio of their molecules rms speeds is equal to

(A) unity. The molecules have the same rms speed.
(B) the ratio of their masses.
(C) inverse ratio of the square root of their masses.
(D) inverse ratio of their masses.
(E) ratio of the square root of their masses.

(ii) Gas molecules in equilibrium have a distribution which is **NOT**

(A) given by the Maxwell Distribution for speed.
(B) has a mean kinetic energy per degree of freedom of kT/2.
(C) symmetric in velocity and asymmetric in speed.
(D) has an rms speed that is proportional to the absolute temperature.
(E) has a lower average than rms speed.

(iii) The critical point of a real gas is

(A) where the gas changes to a liquid.
(B) where the PV isotherm curve goes horizontal.
(C) the temperature which divides the regions where a gas can be made a liquid by applying pressure and where it cannot.
(D)the pressure and volume on the critical temperature isotherm curve where it goes critical.
(E) the point where gas, liquid and solid can coexist in equilibrium.

(iv) Which of these gas laws are incorrectly labeled or stated?

(A) Avogadro's Law: $V \propto n$ for constant temperature and pressure.
(B) Diver's Law $P \propto n$ for constant temperature and volume.
(C) Boyle's Law: $V \propto \frac{1}{P}$ at constant temperature.
(D) Charles's Law: $V \propto T$ for constant pressure.
(E) Gay-Lussac's law: $P \propto T$ for constant volume.
(F) Ideal Gas Law: $PV = nRT = NkT$
(G) Clausias equation of state: $P\left(\frac{V}{n} - b\right) = RT$
(H) van der Waals equaiton of state: $\left(p + \frac{a}{(V/n)^2}\right)\left(\frac{V}{n} - b\right) = RT$
(I) Dalton's law: for two gases mixed together and in equilibrium $P_1 + P_2 = P_{total} = (n_1 + n_2)RT$

(v) What temperature is used for ideal gas law and heat engine calculations and formula:

(A) absolute temperature.
(B) °C = Centigrade.
(C) K Kelvin.
(D) °F = Farhenheit.
(E) R = Rankine.

(d) [5 points] Circle correct answer (i) You have been running track races in smooth-soled

shoes. During each start, you have been wasting 100 joules of energy as thermal energy because of friction between your shoes and the track. To help this situation, you purchase a pair of spiked shoes. Now when you start a race, the frictional force your feet experience from the track is increased by a factor of 5 and the shoes do not slide across the track at all. During each start, the amount of energy you now waste as thermal energy because of friction between your spiked shoes and the track is

(A) 0 joules.
(B) 4 joules.
(C) 500 joules.
(D) 20 joules.
(E) not enough information to determine the change.

(ii) If the The total energy of a rubber ball in a box is contained in the ball's gravitational potential energy, its kinetic energy of motion, and its thermal energy. Energy can be transferred from one of these forms to another as the ball moves around. You throw the ball into the box and leave it for 10 minutes. When you return, most of the ball's energy will have

(A) turned into thermal energy.
(B) turned into gravitational potential energy.
(C) turned into kinetic energy of motion.
(D) turned into random bouncing of the ball around the box.
(E) shared equally into all forms of energy.

(iii) Which of these is **not** a latent heat?

(A) heat of fusion.
(B) heat of vaporization.
(C) heat of phase change.
(D) heat of friction
(E) none of the above
vspace0.2in

(iv) A soft drink is rated to have 160 Calories (=160 kilocalories). What is the equivalent energy in joules?

(A) 669 joules.
(B)6.69×10^3.
(C) 6.69×10^4.
(D) 6.69×10^5.
(E) 6.69×10^6.

(v) At standard temperature (273 K) and pressure (1 atm) how many air molecules are there per cubic cm?

(A) 269 molecules/cm^3.
(B) 2.69×10^{19} molecules/cm^3.
(C) 2.69×10^{23} molecules/cm^3.
(D) 6.02×10^{23} molecules/cm^3.
(E) 2.69×10^{25} molecules/cm^3.

(e) [5 points] Circle correct answer

(i) Which of these is **not** a possible efficiency for a heat engine working between to heat reservoirs at 127° and 27°C?

(A) zero.
(B) 0.1
(C) 0.2
(D) 0.25
(E) 0.3
(F) all above are acceptable

(ii) What is the change in entropy of 1 kg of water when it is heated from 0°C to 100°C?

(A) 10 cal/K
(B) 100 cal/K
(C) 1000 ln(373/273) cal/K
(D) 1000 cal/K
(E) 1000 ln(273/373) cal/K

(iii) The critical point of a real gas is

(A) where the gas changes to a liquid.
(B) where the PV isotherm curve goes horizontal.
(C) the temperature which divides the regions where a gas can be made a liquid by applying pressure and where it cannot.
(D)the pressure and volume on the critical temperature isothem curve where it goes critical.
(E) the point where gas, liquid and solid can coexist in equilibrium.

(iv) The work done by an ideal gas that adiabatically expands from pressure P_1 and volume V_2 to pressure P_1 and volume V_2 is given by the expression

(A) $P_1V_1^\gamma - P_2V_2^\gamma$.
(B) $(P_1V_1 - P_2V_2)^\gamma$.
(C) $(P_1V_1 - P_2V_2)/(\gamma - 1)$.
(D) $(P_1V_1 - P_2V_2)^{1/\gamma}$.
(E) $P_1(V_1 - V_2)$.

(v) If the entropy of a substance is proportional to its absolute temperature, T, then its heat capacity must be proportional to

(A) 1/T.
(B) constant (T^0)
(C) T.
(D) T^2.
(E) T^3.

2. [15 points] You are a scientist or engineer working at the Admunsen-Scott South Pole station. Staying warm enough to survive is a priority. The ice surface and air temperature is roughly -45° C.

(a) [4 points] The bottom of each of your boots is about 2500 cm^2. If they are 3 cm thick made of insulating plastic with thermal conductivity of about 10^5 kcal/(s*m*°C), how much heat is lost through the soles of your boots?

(b) [4 points] You go outside when the Sun is shining. Your effective cross-sectional area is about 0.7 m^2. What is your heat gain from the Sun?

(c) [2 points] Why does it matter whether the wind is blowing or not?

(d) [1 points] Does it matter if you are moving?

(e) [4 points] What about breathing air in and out? A typical breathing rate is roughly 6 liters per minute (about a quarter of a mole of air per minute) what heat loss rate does that correspond to? Does it matter that you breath freezes on your mask? Does that cause heat loss?

3. [20 points] Consider the following three-step process cycle: Heat is allowed to flow out of an ideal monatomic gas at constant volume so that its pressure drops from 2.2 atm to 1.5 atm. Then the gas expands at constant pressure, from a volume of 6.8 L to 10.0 L, where the temperature reaches its original value. The gas then moves along the isotherm back to its starting point.

(a) [4 points] Draw a PV diagram showing the three process cycle.?

(b) [4 points] Calculate the total work done by the gas in the process cycle.

(c) [4 points] Calculate the change in internal energy of the gas in the first two steps (processes) of the cycle.

(d) [4 points] What is heat flow into or out of the gas on each of the three steps?

(e) [4 points] Is this a heat engine or a refrigerator? What is its efficiency or coefficient of performance? Explain your answer.

(f) [2 points] What is the entropy change for the whole world in one complete cycle? Why?

4. [10 points] At a crime scene, the forensic investigator notes that the 8.2-g lead bullet that was stopped in a door-frame melted completely on impact. Assume the bullet was fired at room temperature (20°C). The heat of fusion for lead is 5.9 kcal/kg and its melting point is 327°C. The specific heat of lead is 0.031 cal/g*°C or 130 J/kg °C

(a) [3 points] What is the heat required to get to the melting point?

(b) [3 points] What is the heat required for melting the bullet?

(c) [2 points] What is the energy in joules required to heat and melt the bullet?

(d) [2 points] What does the investigator calculate was the *minimum* muzzle velocity of the gun?

5. [15 points] You plan to have iced tea for your afternoon party. You have an insulated container with 3 kg of tea (essentially water) at 20°C, to which you add ice at -10°C.

(a) [5 points] How much ice at -10°C do you need to add to the 20°C tea in order to have a resulting mixture of tea (water) and 0.2 kg of ice in equilibrium? What is the equilibrium temperature?

(b) [5 points] Suppose the container of the iced tea is left in the state equilibrium describe in part (a) and the temperature outside the container is fixed at 20°C. Suppose further that the container has a total surface area of 2500 cm^2, a thickness of 1 cm and is made of material with thermal conductivity 0.025 V/m-°C. While there is still ice left, find the rate at which heat energy is entering the container and how long it takes the ice to melt.

(c) [5 points] What was the entropy change in part (b) and in part (a)?

6. [15 points] After learning about heat engines and pumps, your class mate decides to start a company to develop, build, sell, and install systems that use a heat pump to cool or heat houses and buildings that uses a heat engine to produce the power to run the heat pump. You classmate claims that such a system will save the user a lot of money compared to conventional systems or even a heat pump.

Consider a typical California house where the temperature difference between inside and outside is rarely more than 10°C. Because of windows, doors, cracks and other leaks, at a 10°C temperature difference heat leaks into or out of the house at the rate of about 2500 watts.

(a) [2 points] Natural gas used for heating costs about $ 0.75 per therm. A therm of gas is 10,000 BTU (British thermal units) and there are 3413 BTU per kilowatt-hour. (2.930 KW-hr per therm). What is the cost per hour and per month (30 days) to heat that house with natural gas? A typical gas heater has 50% efficiency as a lot of the heat goes out the flue.

(b) [2 points] What does it cost in dollars per hour and per month (30 days) to heat that house with electricity? Assume electricity costs $ 0.1/KW-hr.

(c) [4 points] What would it cost with an electrically powered heat pump to heat or cool that house with the 10° temperature difference? With a heat pump that was at 50% of maximum coefficient of performance?

(d) [2 points] Draw a diagram that shows your friends proposed system and label the components of the system and the energy flows.

(e) [4 points] Assuming that the heat engine works at an astounding 90% of its maximum efficiency and the heat pump works with an amazing 80% of its maximum performance, what does it cost to make up the difference with electrical power.?

(f) [1 points] If the heat pump costs 2000$ per unit according to the business plan, should you invest in your fellow student's new business as a venture capitalist?

(g) [1 points] If a heat pump system costs about 1500 dollars, a gas furnace 1000 dollars, and an electric heater system 500 dollars, which should you purchase if you need a new heating system? Hint: What would be your total cost for each of the three cases at the end of the first or second year of ownership?

End of Examination

7. [15 points] Nitrogen N_2) molecules have a molecular weight of 28 and a radius of 2×10^{-10}m. Numerically estimate the following quantities and indicate how each depend on temperature and pressure:

(a) [5 points] the mean free path at Standard Temperature and Pressure (STP)

(b) [5 points] The average time between collisions of any one molecure with any of the others at STP

(c) [5 points] The specific heat at constant volume and constant pressure at STP.

8. [15 points] A Carnot engine working on a satellite in outer space has to deliver a fixed amount power at the rate $\dot{W}$. The temperature of the heat source is also fixed, at T_1. The lower temperature reservoir at temperature T_2 consists of a large body of area A; its temperature is maintained at T_2 because it radiates energy into space as much heat as is delivered to it by the engine. The Carnot engine has to be designed that, for a given $\dot{W}$ and T_1, A has a minimum value. Show that A has a minimum value when T_2 takes the value of $3T_1/4$.

Solutions

1. [25 points] Short Questions

(a) [5 points] Circle T or F for True or False

T F (i) The heat capacity of a body is the amount of heat that it can store at a given temperature.

T F (ii) If the absolute temperature of a body triples, the rate at which it radiates energy increases by a factor of 9.

T F (iii) When a system goes from equilibrium state 1 to state 2, the change in the internal energy is the same for all processes.

T F (iv) The internal energy of a given amount of an ideal gas at equilibrium depends only on its absolute temperature.

T F (v) For any material that expands when heated, C_P is greater than C_V.

T F (vi) Pressure and Temperature are state variables.

T F (vii) For a solid the mean distance between atoms is 2×10^{-10} m.

T F (viii) The Carnot efficiency changes by the same amount from either increasing the hotter temperature reservoir by 10% or decreasing the lower reservoir temperature by 10% to within 1%.

T F (ix) A good emitter of radiation is a good absorber of radiation.

T F (x) According to the principle of equipartition, energy is shared equally among the active degrees of freedom in an amount of $\frac{1}{2}kT$ on average.

(b) [5 points] Circle correct answer

(i) A scuba diver releases a spherical air bubble 3-cm diameter at a depth of 14-m in water at 298 K. Approximately what is the air bubble's radius just as it reaches the surface, if the temperature stays constant. Take the density of water to be 1000 kg/m^3.

(A) 3 cm.

(B) 4 cm.

(C) 5 cm..

(D) 6 cm

(E) 7 cm.

Answer: (A) Why: $PV = P_0V_0 = nkT$ $P_0 = 15m \times 1000 \times 9.8 + 1.013^5$N/m^2 $= 2.48300 N/m^2$ $P = 1.013 \times 10^5 N/m^2$ $V = V_0 \times 2.48/1.013 = 2.45114 V_0 = 2.45114\pi(3cm)^3/4 = \pi r^3/4$ $r = 3.0 * (2.45114)^{1/3} = 4.04492cm$

(ii) You are on a trip overseas and it seems very cold. You use a local thermometer to see how cold it really is It reads -40°C, what temperature is that in ° F? (A) -104° F.

(B) -60° F.

(C) -40° F.

(D) -20° F.

(E) -10° F..

Answer: (C) -40° F..

Why: $X°F = 9Y°C/5 + 32°F = 9 \times -40/5 - 32 = -40°F$

(iii) A 1 meter radius balloon containing an ideal gas is cooled from 400 K to 300 K. How much does its volume and surface area change assuming the external pressure is constant?

(A) decrease by 33% and 21%.

(B) decrease by 25% and 17%.

(C) no change.

(D) increase by by 25% and 17%.

(E) increase by 33% and 21%.

Answer: (B) decrease by 25% and 17%.

Why: We use the ideal gas law $PV = nRT$ which we rewrite as $V = (nR/P)T$. That means the ratio of volumes is $V_{300}/V_{400} = 300/400 = 0.75$ which is a 25% decrease in volume. The area is $4\pi R^2$ and the volume is $4\pi/3\ R^3$ so that the area ratios will be the 2/3rds power of the volume ratios.

(iv) A 1 meter radius glass sphere is heated by 100°C. About how much does its surface area increase, given $\alpha = 9 \times 10^{-6}/°$C?

(A) increase by 3 cm^2.

(B) increase by 50 cm^2.

(C) increase by 100 cm^2.

(D) increase by 200 cm^2

(E) increase by 300 cm^2

Answer: (D) increase by 200 cm^2

Why: Radius will increase by $R = R_o(1 + \alpha\Delta T) = 1m * (1 + 9 \times 10^{-6} \times 100) = 1.0009m$ Change in area is $A - A_o = 4\pi(R^2 - R_o^2) = 4\pi(1.0009^2 - 1)m^2 = 0.0226m^2 = 226cm^2$.

(v) At standard temperature (273 K) and pressure (1 atm) how many air molecules are there per cubic cm?

(A) 269 molecules/cm^3.

(B) 2.69×10^{19} molecules/cm^3.

(C) 2.69×10^{23} molecules/cm^3.

(D) 6.02×10^{23} molecules/cm^3.

(E) 2.69×10^{25} molecules/cm^3.

Answer: (B) 2.69×10^{19} molecules/cm^3.

Why: We can use the ideal gas law $PV = nRT$ and that there are Avogadro's Number 6.024×10^{23} molecules per mole. $V = 1 \times 8.315 \times 273/1.013 \times 10^5 = 0.0224 m^3 = 22.4 liters = 22400 cm^2$ Thus the molecule number density is $N = 6.02 \times 10^{23}/22400 cm^2 = 2.69 \times 10^{19}$

(c) [5 points] Circle correct answer

(i) Two gases are mixed and in equilibrium, the ratio of their molecules rms speeds is equal to

(A) unity. The molecules have the same rms speed.
(B) the ratio of their masses.
(C) inverse ratio of the square root of their masses.
(D) inverse ratio of their masses.
(E) ratio of the square root of their masses.
Answer: (C) inverse ratio of the square root of their masses.
Why: $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\rho}{P}}$ so that the ratio of the speeds is $v_1/v_2 = \sqrt{m_2/m_1}$

(ii) Gas molecules in equilibrium have a distribution which is **NOT**

(A) given by the Maxwell Distribution for speed.
(B) has a mean kinetic energy per degree of freedom of kT/2.
(C) symmetric in velocity and asymmetric in speed.
(D) has an rms speed that is proportional to the absolute temperature.
(E) has a lower average than rms speed.
Answer: (D) have an rms speed that is proportional to the absolute temperature.
Why: $v_{rms} = \sqrt{\frac{3kT}{m}}$

(iii) The critical point of a real gas is

(A) where the gas changes to a liquid.
(B) where the PV isotherm curve goes horizontal.
(C) the temperature which divides the regions where a gas can be made a liquid by applying pressure and where it cannot.
(D)the pressure and volume on the critical temperature isotherm curve where it goes critical.
(E) the point where gas, liquid and solid can coexist in equilibrium.
Answer: (D)the pressure and volume on the critical temperature isothem curve where it goes critical.

Why: The point where gas, liquid and solid can coexist in equilibrium is called the **triple point**. The temperature which divides the regions where a gas can be made a liquid by applying pressure and where it cannot is called the **critical temperature**. The PV isotherm curve goes horizontal for all temperatures below the critical temperature. The gas can change to a liquid along a line of pressure and temperatures.

(iv) Which of these gas laws are incorrectly labeled or stated?

(A) Avogadro's Law: $V \propto n$ for constant temperature and pressure.
(B) Diver's Law $P \propto n$ for constant temperature and volume.
(C) Boyle's Law: $V \propto \frac{1}{P}$ at constant temperature.
(D) Charles's Law: $V \propto T$ for constant pressure.
(E) Gay-Lussac's law: $P \propto T$ for constant volume.
(F) Ideal Gas Law: $PV = nRT = NkT$
(G) Clausias equation of state: $P\left(\frac{V}{n} - b\right) = RT$

(H) van der Waals equaiton of state: $\left(p + \frac{a}{(V/n)^2}\right)\left(\frac{V}{n} - b\right) = RT$

(I) Dalton's law: for two gases mixed together and in equilibrium $P_1 + P_2 = P_{total} = (n_1 + n_2)RT$

Answer: (I) Dalton's law: for two gases mixed together and in equilibrium $P_1 + P_2 = P_{total} = (n_1 + n_2)RT$

Why: In the second portion equality the units of P and nRT are different by V.

(v) What temperature is used for ideal gas law and heat engine calculations and formula:

(A) absolute temperature.
(B) °C = Centigrade.
(C) K Kelvin.
(D) °F = Farhenheit.
(E) R = Rankine.

Answer: (A) absolute temperature.

(d) [5 points] Circle correct answer

(i) You have been running track races in smooth-soled shoes. During each start, you have been wasting 100 joules of energy as thermal energy because of friction between your shoes and the track. To help this situation, you purchase a pair of spiked shoes. Now when you start a race, the frictional force your feet experience from the track is increased by a factor of 5 and the shoes do not slide across the track at all. During each start, the amount of energy you now waste as thermal energy because of friction between your spiked shoes and the track is

(A) 0 joules.
(B) 4 joules.
(C) 500 joules.
(D) 20 joules.
(E) not enough information to determine the change.

Answer: (A) 0 joules.

Why: Only sliding friction turns ordered energy into thermal energy. Since your spiked shoes prevent your feet from sliding on the track, you waste no energy at all as thermal energy due to friction between your shoes and the track.

(ii) If the The total energy of a rubber ball in a box is contained in the ball's gravitational potential energy, its kinetic energy of motion, and its thermal energy. Energy can be transferred from one of these forms to another as the ball moves around. You throw the ball into the box and leave it for 10 minutes. When you return, most of the ball's energy will have

(A) turned into thermal energy.
(B) turned into gravitational potential energy.
(C) turned into kinetic energy of motion.
(D) turned into random bouncing of the ball around the box.
(E) shared equally into all forms of energy.

Answer: (A) turned into thermal energy.

Why: After a while, the ball will stop bouncing. It will have as little gravitational potential energy as possible, because it will be in the bottom of the box. It won't be moving, so its kinetic energy will be zero. And it won't be hopping around the box, so it won't have any random bouncing. Instead, all of its energy will have become thermal energy.

(iii) Which of these is **not** a latent heat?

(A) heat of fusion.
(B) heat of vaporization.
(C) heat of phase change.
(D) heat of friction
(E) none of the above

Answer: (D) heat of friction

Why: latent heat is the amount of energy released in a change of phase. Heat of friction is the conversion of motional energy into heat energy.

(iv) A soft drink is rated to have 160 Calories (=160 kilocalories). What is the equivalent energy in joules?

(A) 669 joules.
(B)6.69×10^3.
(C) 6.69×10^4.
(D) 6.69×10^5.
(E) 6.69×10^6.
Answer: (D) 6.69×10^5.
Why: 1 Calorie = 4.18×10^3 joules.

(v) At standard temperature (273 K) and pressure (1 atm) how many air molecules are there per cubic cm?
(A) 269 molecules/cm^3.
(B) 2.69×10^{19} molecules/cm^3.
(C) 2.69×10^{23} molecules/cm^3.
(D) 6.02×10^{23} molecules/cm^3.
(E) 2.69×10^{25} molecules/cm^3.
Answer: (B) 2.69×10^{19} molecules/cm^3.
Why: We can use the ideal gas law $PV = nRT$ and that there are Avogadro's Number 6.024×10^{23} molecules per mole. $V = 1 \times 8.315 \times 273/1.013 \times 10^5 = 0.0224m^3 = 22.4liters = 22400cm^2$ Thus the molecule number density is $N = 6.02 \times 10^{23}/22400cm^2 = 2.69 \times 10^{19}$

(e) [5 points] Circle correct answer
(i) Which of these is **not** a possible efficiency for a heat engine working between to heat reservoirs at 127° and 27°C?
(A) zero.
(B) 0.1
(C) 0.2
(D) 0.25
(E) 0.3
(F) all above are acceptable
Answer: (E) 0.3
Why: $\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{27+273}{127+273} = 1 - \frac{300}{400} = 0.25$ is the Carnot efficiency and no heat engine can exceed this efficiency.

(ii) What is the change in entropy of 1 kg of water when it is heated from 0°C to 100°C?
(A) 10 cal/K
(B) 100 cal/K
(C) 1000 ln(373/273) cal/K
(D) 1000 cal/K
(E) 1000 ln(273/373) cal/K
Answer: (C) 1000 ln(373/273) cal/K
Why: $dS = dQ/T$ so $\Delta S = \int \frac{dQ}{T} = \int \frac{mC_p dT}{T} = mC_p ln(T_f/T_i)$ $T_f = 273 + 100 = 373$K. $T_i = 273$K

(iii) The critical point of a real gas is
(A) where the gas changes to a liquid.
(B) where the PV isotherm curve goes horizontal.
(C) the temperature which divides the regions where a gas can be made a liquid by applying pressure and where it cannot.
(D)the pressure and volume on the critical temperature isothem curve where it goes critical.
(E) the point where gas, liquid and solid can coexist in equilibrium.
Answer: (D)the pressure and volume on the critical temperature isothem curve where it goes critical.
Why: The point where gas, liquid and solid can coexist in equilibrium is called the **triple point**. The temperature which divides the regions where a gas can be made a liquid by applying pressure and where it cannot is called the **critical temperature**. The PV isotherm curve goes horizontal for all temperatures below the critical temperature. The gas can change to a liquid along a line of pressure and temperatures.

(iv) Which of these gas laws are incorrectly labeled or stated?
(A) Avogadro's Law: $V \propto n$ for constant temperature and pressure.
(B) Diver's Law $P \propto n$ for constant temperature and volume.
(C) Boyle's Law: $V \propto \frac{1}{P}$ at constant temperature.
(D) Charles's Law: $V \propto T$ for constant pressure.
(E) Gay-Lussac's law: $P \propto T$ for constant volume.

(F) Ideal Gas Law: $PV = nRT = NkT$
(G) Clausias equation of state: $P\left(\frac{V}{n} - b\right) = RT$
(H) van der Waals equaiton of state: $\left(p + \frac{a}{(V/n)^2}\right)\left(\frac{V}{n} - b\right) = RT$
(I) Dalton's law: for two gases mixed together and in equilibrium $P_1 + P_2 = P_{total} = (n_1 + n_2)RT$
Answer: (I) Dalton's law: for two gases mixed together and in equilibrium $P_1 + P_2 = P_{total} = (n_1 + n_2)RT$
Why: In the second portion equality the units of P and nRT are different by V.

(v) What temperature is used for ideal gas law and heat engine calculations and formula:
(A) absolute temperature.
(B) °C = Centigrade.
(C) K Kelvin.
(D) °F = Farhenheit.
(E) R = Rankine.
Answer: (A) absolute temperature.

2. [15 points] You are a scientist or engineer working at the Admunsen-Scott South Pole station. Staying warm enough to survive is a priority. The ice surface and air temperature is roughly -45° C while your feet are at about 34°C.
(a) [4 points] The bottom of each of your boots is about 250 cm^2. If they are 3 cm thick made of insulating plastic with thermal conductivity of about 10^5 kcal/(s*m*°C), how much heat is lost through the soles of your boots?
$\frac{dQ}{dt} = kA\frac{T_1 - T_2}{l} = 10^{-5} \times 2 \times 250 \times (34 + 45)/0.03 = 2 \times 6.58 cal/sec = 2 \times 27.5 watts$
(b) [4 points] You go outside when the Sun is shining. Your effective cross-sectional area is about 0.7 m^2. What is your heat gain from the Sun on your clothes assuming their emissivity is about 0.5?
$\frac{dQ}{dt} = e\sigma AT^4\Omega = 0.5 \times 5.67 \times 10^{-8} \times 0.7 \times 6000^4 W \times \Omega = 153W$ or use the solar constant

$$\frac{dQ}{dt} = 1000W/m^2 \times eAcos\theta = 1000 \times 0.5 \times 0.7 \times 1 = 350W$$

(b) [2 points] Why does it matter whether the wind is blowing or not?

Convection is very effective in removing heat. Still air has very low thermal conductivity. However, when it is moving, it can transport heat away from a surface more more significantly.

(d) [1 points] Does it matter if you are moving?

Your motion relative to the air results in convection.

(e) [4 points] Does it matter that you breath freezes on your mask? Does that cause heat loss? What about breathing air in and out? A typical breathing rate is roughly 6 liters per minute (about a quarter of a mole of air per minute). What heat loss rate does that correspond to?

No because the latent heat of freezing is not relevant unless you are in thermal contact with the mask. You do breathe air in an out resulting in a heat loss. That heat loss from breathing is roughly estimated as follows: At STP there are 22.4 liters of gas per mole so that rate is roughly 0.27 moles / minute. The heat capacity of a diatomic gas is C = 2.5 R per mole. The temperature change is $\Delta T = 37 + 45 = 82°C$. The heat loss rate is then $\dot{Q} = \dot{n}C\Delta T = (0.27/60)2.5 \times 8.315 \times 82 = 7.67059 cal/sec = 32 watts$

3. [20 points] Consider the following three-step process cycle: Heat is allowed to flow out of an ideal monatomic gas at constant volume so that its pressure drops from 2.2 atm to 1.5 atm. Then the gas expands at constant pressure, from a volume of 6.8 L to 10.0 L, where the temperature reaches its original value. The gas then moves along the isotherm back to its starting point.

(a) [4 points] Draw a PV diagram showing the three process cycle.?

(b) [4 points] Calculate the total work done by the gas in the cycle.

The work done by the gas in the cycle is the enclosed area of the PV diagram. The work done in the isothermal part can be calculated using the ideal gas law. $PV = nRT$ which for constant temperature becomes $PV = P_0V_0 = 1.5atm \times 10liters = 1.5 \times 1.013 \times 10^5 \times 10 \times 10^3 \times 10^{-6} joules = 1.51950 \times 10^3 joules$

$W_3 = \int PdV = \int P_0V_0dV/V = P_0V_0ln(V_1/V_2) = -1.51950 \times 10^3 ln(10/6.8) joules = -586 joules$

No work is done on leg 1 as the volume does not change. $W_1 = 0$ On leg two the volume changes and the pressure is constant. The gas does work $W_2 = P\Delta V = P \times (10-6.8) liters = 1.5 \times 1.013 \times 10^5 \times 3.2 \times 10^3 \times 10^{-6} joules = 486.24 joules$

The net work done is $DeltaW = W_1 + W_2 + W_3 =$ -99.6 joules

(c) [4 points] Calculate the change in internal energy of the gas in the first two steps (processes) of the cycle. For the first two processes together we can realize that they begin and end at the same temperature. That means that the net change in internal energy is zero. That also makes sense when we realize that in a complete cycle we come back to the same internal state and energy so that the net around the cycle is zero and that an isothermal process does not change the internal energy.

Should we want to tackle this more directly calculating the change in internal energy per process (1) and (2) we find we need to know the number of degrees of freedom (DOF) for the ideal gas. For the first process we could use the ideal gas law $PV = nRT$ and realize that at constant volume the change in absolute temperature is proportional to the change in absolute pressure. Since the internal energy is proportional to the temperature $U = \frac{DOF}{2}RT$ that means that the internal energy changes by an amount proportional to the change in the pressure. Namely $\Delta U_1 = \frac{DOF}{2}RT = \frac{DOF}{2}\Delta PV = \frac{DOF}{2}(2.2 - 1.5) \times 1.013 \times 10^5 \times 6.8 \times 10^{-3} joules = \frac{DOF}{2}482.2 joules$

Or we can use the approach that the change in internal energy around the whole cycle is zero and that for the isothermal process (3) no temperature change means $\Delta U_3 = 0$.

For process 2 - constant pressure, the work done is $W_2 = P\Delta V$ $dU = dQ - dW$ or $\Delta U = \Delta Q - \Delta W$

(d) [4 points] What is heat flow into or out of the gas on each of the three steps?
(1) $\Delta Q_1 = -\frac{DOF}{2}482.2 joules$
(2) $\Delta Q_2 = \frac{DOF}{2}482.2 joules + W_2 = \frac{DOF}{2}482.2 joules + 486.2 joules$
(3) $\Delta Q_1 = -W_3 = 586 joules$
$dU = dQ - dW$ or $\Delta U = \Delta Q - \Delta W$

(e) [4 points] Is this a heat engine or a refrigerator? What is its efficiency or coefficient of performance? Explain your answer.

It is a poor hea tpump (refrigerator) . It takes heat from the colder reservoir and rejects heats to the warm

(f) [2 points] What is the entropy change for the whole world in one complete cycle? Why?

4. [10 points] At a crime scene, the forensic investigator notes that the 8.2-g lead bullet that was stopped in a door-frame melted completely on impact. Assume the bullet was fired at room temperature (20°C). The heat of fusion for lead is 5.9 kcal/kg and its melting point is 327°C. The specific heat of lead is 0.031 cal/g*°C or 130 J/kg °C

(a) [3 points] What is the heat required to get to the melting point?

Heat $= 8.2g \times (327 - 20)°C \times 0.031 cal/g*$°C $= 78$ cal

(b) [3 points] What is the heat required for melting the bullet?

Heat melting $= 8.2g \times 5.9 \times 10^3 \times 10^{-3} = 48.4$ cal

(c) [2 points] What is the energy in joules required to heat and melt the bullet?

Energy $= 4.18 \times (78 + 48.4)$ j $= 528$ joules

(d) [2 points] What does the investigator calculate was the *minimum* muzzle velocity of the gun?

KE $= \frac{1}{2}mv^2 = 528$ joules. $v = \sqrt{2 \times 528/8.2 \times 10^{-3}} = 359m/s$

5. [15 points] You plan to have iced tea for your afternoon party. You have an insulated container with 3 kg of tea (essentially water) at 20°C, to which you add ice at -10°C.

(a) [5 points] How much ice at -10°C do you need to add to the 20°C tea in order to have a resulting mixture of tea (water) and 0.2 kg of ice in equilibrium? What is the equilibrium temperature?

The equilibrium temperature will be 0°. Thus the ice will warm and all but 0.2 kg will melt.

$Q_{water} = m_{water}C_P\Delta T_{water} = 3000gm \times 1cal/gm -^\circ C \times -20°C = -6 \times 10^4 cal$

$Q_{ice} = m_{ice}C_P\Delta T_{ice} + (m_{ice} - 200gm)L_H = m_{ice} \times 1cal/gm *^\circ C \times 10°C + (m_{ice} - 200gm) \times 80cal/gm = (10 + 80)m_{ice}cal/gm + 1.6 \times 10^4 cal$

The two heats have to sum to zero so we have $(10+80)m_{ice}cal/gm - 1.6 \times 10^4 cal = 6 \times 10^4 cal$ or $90m_{ice} = 7.6 \times 10^4 gm$ yielding $m_{ice} = 844.444gm = 0.844444kg$

(b) [5 points] Suppose the container of the iced tea is left in the state equilibrium describe in part (a) and the temperature outside the container is fixed at 20°C. Suppose further that the container has a total surface area of 2500 cm^2, a thickness of 1 cm and is made of material with thermal conductivity 0.025 W/m-°C. While there is still ice left, find the rate at which heat energy is entering the container and how long it takes the ice to melt.

We use Newton's heat conductivity equation

$\frac{dQ}{dt} = -\kappa A\frac{dT}{dx} = 0.025W/m -^\circ C \times 2500cm^2 \times \frac{20^\circ c}{1cm} = 0.025 \times 25 \times 20watts = 12.5watts = 3calories/second$

The heat needed to melt the ice is $Q = m_{ice}L_H = 200gm \times 80cal/gm = 16000cal$ At three calories per second that takes a time $t = 16000/3sec = 5350.4sec = 89.2minutes = 1.486hours$.

(c) [5 points] What was the entropy change in part (b) and in part (a)?

The entropy change in part (b) is very easy to calculate: $\Delta S = \Delta Q/T = 16000cal/273K = 58.608cal/K$ for the ice water. and for the surroundings $\Delta S = \Delta Q/T = -16000cal/293K = -54.608cal/K$ The net result is $\Delta S = 4cal/K$

The entropy change in part (a) is much more complicated to calculate. $dS = dQ/T$ or $\Delta S = \int dQ/T$

For the water $\Delta S = \int mC_p dT/T = mC_P ln(T = 273 + 0°C/T = 273 + 20°C)3000 \times 1cal/K ln(273/293) = -212.102cal/K$ For the ice $\Delta S = 844.444gm \times 1cal/gm - K ln(273/263) + (844.444gm - 200gm) \times 80cal/gm/273K = 31.513 + 188.848cal/K = 220.361cal/K$ The total is then $\delta S = 8.259cal/K$.

6. [15 points] After learning about heat engines and pumps, your class mate decides to start a company to develop, build, sell, and install systems that use a heat pump to cool or heat houses and buildings that uses a heat engine to produce the power to run the heat pump. You classmate claims that such a system will save the user a lot of money compared to conventional systems or even a heat pump.

Consider a typical California house where the temperature difference between inside and outside is rarely more than 10°C and we take the indoor temperature as 22° C (71.6° F). Because of windows, doors, cracks and other leaks, at a 10°C temperature difference heat leaks into or out of the house at the rate of about 2500 watts.

(a) [2 points] Natural gas used for heating costs about \$ 0.75 per therm. A therm of gas is 100,000 BTU (British thermal units) and there are 3413 BTU per kilowatt-hour. (29.30 KW-hr per therm). What is the cost per hour and per month (30 days) to heat that house with natural gas? A typical gas heater has 50% efficiency as a lot of the heat goes out the flue.

The heat rate needed from the natural gas is $dQ/dt = 2500w/0.5 = 5000watts$ The number of therms per hour is then $dGas/dt = 5000W/29.30KW-hrpertherm = 0.1706therms/hr$ at 0.75\$ per therm this is Burn Rate = 0.1706×0.75\$ /hr = 0.1280 dollars/hr or about 90 dollars per month if the furnace were on all the time.

(b) [2 points] What does it cost in dollars per hour and per month (30 days) to heat that house with electricity? Assume electricity costs \$ 0.1/KW-hr.

The cost rate for electrical heating would be $d\$\ dt = 2500watts \times 0.1\$\ /kw - hr =$ \$ $0.25/hour$ or about \$ 180 / month.

(c) [3 points] What would it cost with an electrically powered heat pump to heat or cool that house with the 10° temperature difference? With a heat pump that was at 50% of maximum coefficient of performance?

The maximum coefficient of performance $CP = |Q_H|/|W| = T_L/(T_H - T_L) = 295/10 = 29.5$. Thus this heat pump has CP = 14.75. The energy rate needed to pump 2500 watts to the warmer heat reservoir is $dQ/dt = 2500watts/14.75 = 169.492watts$. The electrical cost per hour would be $169.492watts \times 0.1dollars/hr = 0.01695dollars/hr$ and 12.20 dollars per month.

(d) [2 points] Draw a diagram that shows your friends proposed system and label the components of the system and the energy flows.

(e) [4 points] Assuming that the heat engine works at an astounding 90% of its maximum efficiency and the heat pump works with an amazing 80% of its maximum performance, what does it cost to make up the difference with electrical power? State you approach to the problem clearly.

If the heat engine and the heat pump worked at the same efficiency, then they would just cancel each other out. That is the heat engine would put as much heat into the lower reservoir as the heat pump took out and vice versa, when the power output of the heat engine was used to power the heat pump. To get the heating or cooling required one would need to add electrical power and just use the coefficient of performance of the heat pump to pump out the heat required.

We want the net result to be the addition of heat (for heating) at a rate 2500 watts higher than being removed by the heat engine. For the hotter reservoir we have

$$\dot{Q}_{H-net} = 2500 watts = \dot{Q}_{H-pump} - \dot{Q}_{H-engine}$$

For the lower temperature reservoir we have $\dot{Q}_{L-pump}$ and $\dot{Q}_{L-engine}$ for the heat flow rates. Conservation of energy gives us for the engine and pump: $W_x = Q_{H-x} - Q_{L-x}$. For Carnot efficiency $W = (T_{H-x} - T_{L-x})Q_{H-x}/T_{L-x}$ With different efficiency we need to scale by the given efficiencies. The heat going to the hotter reservoir from the heat pump is $Q_{H-pump} = \epsilon_{pump}\frac{T_L}{T_H - T_L}W_{pump} = 0.8 \times 29.5 W_{pump} = 23.6 W_{pump}$

The heat taken from the hotter reservoir by the heat engine is $Q_{H-engine} = \frac{T_L}{\epsilon_{engine}(T_H - T_L)}W_{pump} = 29.5/0.9 W_{engine} = 32.77778 W_{engine}$

The difference between these as rates must be 2500 watts $23.6 W_{pump} = 32.77778 W_{engine} + 2500 watts$. The electical power required would be $P_{electrical} = W_{pump} - W_{engine}$.

Combining the two equations and eliminating W_{pump}, we find $P_{electric} = (\frac{32.77778}{23.6} - 1)W_{engine} + \frac{2500 watts}{23.6} = 0.38889 W_{engine} + 105.93220 watts$.

In other words we find that, if the heat engine work must be positive, then it is best to turn it off and the power required is 105.93220 watts which costs 0.01059 dollars an hour or 7.62712 dollars a month.

If the heat engine can be run as a heat pump, then it makes sense to run it as the heat pump and that would cost 76.27 watts, 0.00763 dollars per hour, or \$ 5.49 per month.

(f) [1 points] If the heat pump costs 2000\$ per unit according to the business plan, should you invest in your fellow student's new business as a venture capitalist?

Clearly, the answer is no, since the best solution is to keep the heat engine turned off. If it can work as a heat pump, it might be a good heat pump business depending on the heat pump competition.

(g) [1 points] If a heat pump system costs about 1500 dollars, a gas furnace 1000 dollars, and an electric heater system 500 dollars, which should you purchase if you need a new heating system? Hint: What would be your total cost for each of the three cases at the end of the first or second year of ownership?

If we neglect energy rebates and the interest rate for a year, then

Electric Power heating costs for a year $= 500 + 6 \times 180$ \$ $=$ \$ 1580 and for two years $= 500 + 12 \times 180$ \$ $=$ \$ 2660.

Gas heating cost for a year $= 1000 + 6 \times 90$ \$ $=$ \$ 1540 and for two years $= 1000 + 12 \times 90$ \$ $=$ \$ 2080.

Heat pump heating costs for a year $= 1500 + 6 \times 12.20$ \$ $=$ \$ 1573 and for two years $= 1500 + 12 \times 12.20$ \$ $=$ \$ 1646.40.

For the one period Gas heating is slightly preferred but all are comparable. For longer term horizon the heat pump is least expensive