Lanzara Midterm 2 Prob 1 Soln

Friday, November 07, 2008
10:25 AM

a) Cube will remain moving at initial velocity, v_0

b) $F = ma = -kx$

$$-x = \frac{ma}{k}$$

$$x = -5\,m = 5\,m \text{ compression}$$

c) <u>correct soln</u>

$$\Sigma F_y = 0 \Rightarrow N = mg$$

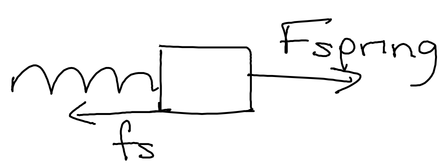
$$f_{s,max} = \mu_s N = \mu_s mg = 1\,N$$

$$ma = (0.2\,kg)(5\,m/s^2) = 1\,N$$

Static Friction provides enough force to give the cube a $5\,m/s^2$ acceleration

spring displacement $= 0$

<u>Alternate soln</u>



$$N = mg$$

$$f_s = \mu_s mg = 1\,N$$

$$ma = F_{spring} - f_s = -kx - f_s$$

$$-x = \frac{ma + f_s}{k} = +10\,m$$

$$x = 10\,m \text{ compression}$$

Solution for Problem 2

1) Before the collision the total momentum is $mv_0 + Mv_0$. The total momentum should be conserved, since there are no external forces in x-direction. Also since there is no force in x-direction acting on the block m (no friction) during the collision, the momentum of the block won't change (just after the collision).

Then we can find the velocities of the block and carts:

the velocity of the block v_0

the velocity of two carts after the collision $Mv_0 = 2Mv_c$ and therefore $v_c = \frac{v_0}{2}$

2) To find the height we can use the conservation of energy. The energy just after the collision (we can't use the initial energy, since the collision is inelastic) is $\frac{mv_0^2}{2} + \frac{2Mv_c^2}{2}$. At the maximum height the relative velocity of the block (respect to carts) is zero, so the block and the carts move as a whole system. To find this velocity we again can use the conservation of momentum (this velocity is not equal to v_c).

$$(m + M)v_0 = (m + 2M)v_f$$

$$v_f = \frac{m + M}{m + 2M} v_0$$

The conservation of energy gives us:

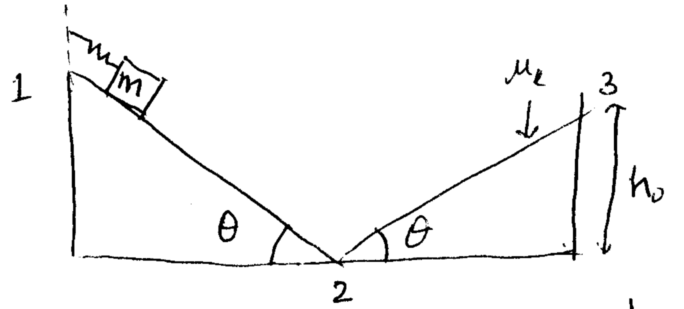
$$\frac{mv_0^2}{2} + \frac{2Mv_c^2}{2} = mgh + \frac{(m + 2M)v_f^2}{2}$$

$$mv_0^2 + 2M\left(\frac{v_0}{2}\right)^2 = 2mgh + \frac{(m + M)^2}{(2M + m)} v_0^2$$

$$h = \frac{Mv_0^2}{4g(2M + m)}$$

SOLUTION

3)



Energy $E_1 = \frac{1}{2} k x_0^2 + \cancel{\frac{g}{2}} Mg h_0$

Energy at 2 $E_2 = \frac{1}{2} m v_2^2$

Energy at 3 $E_3 = Mg h'$

$E_1 - E_2 =$ Work done by friction $= \mu Mg \cos\theta \cdot \dfrac{h_0}{\sin\theta}$ — ①

$E_2 - E_3 =$ Work done by friction on II wedge $= \mu Mg \cos\theta \cdot \dfrac{h'}{\sin\theta}$ — ②

$\cancel{\frac{1}{2} k x_0^2 + Mg}$ ① + ②

$E_1 - E_3 = \mu Mg \cot\theta \,(h_0 + h')$

$\frac{1}{2} k x_0^2 + Mgh_0 - Mgh' = \mu Mg \cot\theta \,(h_0 + h')$

$\frac{1}{2} k x_0^2 + h_0 \,(Mg - \mu Mg \cot\theta) = h' \,(Mg + \mu Mg \cot\theta)$

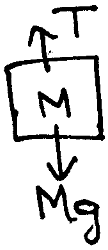
$$h' = \frac{\frac{1}{2} k x_0^2 + h_0 (Mg - \mu Mg \cot\theta)}{(Mg + \mu Mg \cot\theta)}$$

Problem 4 Solution

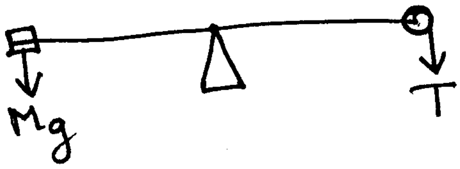
a) Pulley must rotate clockwise because there is no torque to cancel the torque due to the force of tension

$\Rightarrow$ hanging $\boxed{M}$ must accelerate down

$\Rightarrow$ $T < Mg$

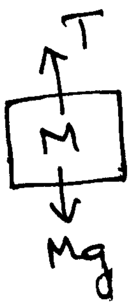


$\Rightarrow$ Rod has angular acceleration



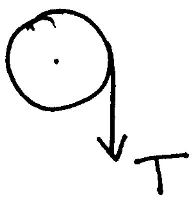
$\therefore$ $\boxed{\text{No equilibrium}}$

b)



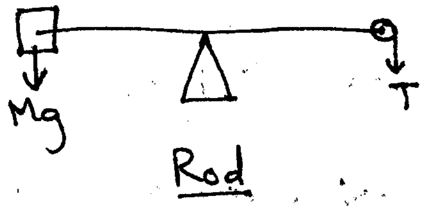
Hanging block

$$\underbrace{Mg - T = Ma}_{\text{Eq } ①}$$ (choosing down as positive)



~~Pulley~~ Spool

$$\underbrace{TR = I\alpha_s}_{\text{Eq } ②}$$ (choosing clockwise as positive)



$$(T-Mg)L = -I_{total}\,\alpha_R$$
$$\approx -ML^2\alpha_R \quad (\because M_{spool} \ll M,\ R \ll L)$$

or $\underbrace{(Mg-T)L = ML^2\alpha_R}_{Eq\ 3}$

(α_R is the magnitude of the angular accel. of the rod + fixed mass + spool system)

Relation between 'a's & α's : $\underbrace{a = \alpha_s R - \alpha_R L}_{Eq\ 4}$

Some algebra with* the 4 equations gives

$$\boxed{T = \frac{Mg}{1+\frac{MR^2}{2I}}}\ , \quad \boxed{a = \frac{g}{1+\frac{2I}{MR^2}}}$$

* If you write Eq 4 as $a = \alpha_s R$ and ignore the motion of the rod, you'll get

$$T = \frac{Mg}{1+\frac{MR^2}{I}}\ , \quad a = \frac{g}{1+\frac{I}{MR^2}}$$