Problem 1

Let T be the tension in the rope between the drum and the pulley, R be the radius of both the drum and the pulley, and $M = 2\,kg$ be the mass of both the drum and the pulley.

$$\Sigma\tau_{drum} = TR = I_{drum}\alpha_{drum} = MR^2\alpha_{drum}$$
$$\Sigma\tau_{pulley} = TR - FR = I_{pulley}\alpha_{pulley} = \frac{1}{2}MR^2\alpha_{pulley}$$
$$F_{pulley} = T + F - Mg = ma_{pulley} = 0$$
$$\alpha_{drum} = -\alpha_{pulley} \Rightarrow TR = MR^2\alpha \text{ and } FR - TR = \frac{1}{2}MR^2\alpha$$
$$TR = 2FR - 2TR \Rightarrow 3T = 2F \Rightarrow T = \frac{2}{3}F$$
$$T + F - mg = 0 \Rightarrow \frac{2}{3}F + F = \frac{5}{3}F = mg \Rightarrow F = \frac{3}{5}mg$$
$$F = \frac{3}{5}(2\,kg)(9.8\,\frac{m}{s^2}) = 11.76\,N$$

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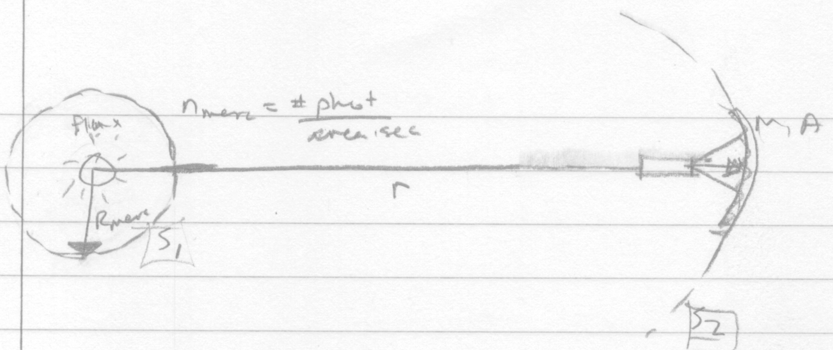
Spelriotopoulos MT2
problem 2.



Surf area of S_1: $4\pi R^2$
S_2: $4\pi r^2$
Area of satellite: $A_{sail} = A$.

a) $F_G = \frac{GMm}{r^2}$ ⟷ F to change dir. of photons $\approx \frac{\Delta p_{phot}}{\Delta t}$ ← perfect reflector: $\Delta p = 2p$ = change in mom. spacecraft.

$\frac{\text{photons}}{\text{area}\cdot\text{sec}} = n_s \Rightarrow$ photons/sec $= n_{merc} \cdot A_{merc\ sphere\ (S_1)} = n_{merc}\, 4\pi R$

At mercury: $\frac{\text{Total momentum}}{\text{sec}} = \frac{\#\text{ photons}}{\text{sec}} \cdot \frac{\text{mom}}{\text{photon}} = 4\pi R_{merc}^2 n_{merc} p$ = total mom/sec through sphere at radius r.

At the large sphere:

$$\frac{\text{mom}}{\text{area}\cdot\text{sec}} = \frac{(\cancel{4\pi} R_{merc}^2 n_{merc} \cdot p)}{\cancel{4\pi} r^2} = \frac{R^2}{r^2} n_{merc} \cdot p$$

$\Rightarrow$ **Momentum hitting the sail per second**: $\frac{R^2}{r^2} n_{merc} \cdot p \cdot A_{sail}$

$\Rightarrow \frac{\Delta p}{\Delta t}$ (change in mom. per sec.):

$$= 2\left(\frac{R^2}{r^2} n_{merc} \cdot p \cdot A\right) = F_{\text{photons}}$$

↑ perfect reflector (see above)

$$\Sigma F = F_{grav} + F_{\text{photons}} = -\frac{GM_s m}{r^2} + \frac{2R^2}{r^2} n_{merc} \cdot p \cdot A.$$

$$\Rightarrow \Sigma \vec{F} = \frac{2R^2 n_{merc} \cdot p \cdot A - GM_s m}{r^2}\ \hat{r}.$$

b) Max attainable speed? Begin $r = R_{merc}$.

$$\tfrac{1}{2}mv_{max}^2 - \tfrac{1}{2}mv_0^2 = \int_{R_{merc}}^{\infty} \vec{F}\cdot d\vec{r} = \int (2pAn_{merc}R_{merc}^2 - GmM_{sun})\frac{1}{r^2}\,dr.$$

$$\Rightarrow \tfrac{1}{2}mv_{max}^2 - \tfrac{1}{2}mv_0^2 = (\qquad)\int \frac{1}{r^2}\,dr = \cancel{-\frac{1}{r}(\ )\Big|_{r=\infty}} + \frac{1}{r}(\qquad)\Big|_{R_{merc}}.$$

Recall $\frac{v_0^2}{r} = \frac{GM_{sun}}{R_{merc}}$ (assuming circular orbit for mercury, to match image / get expression for v_0^2)

p2

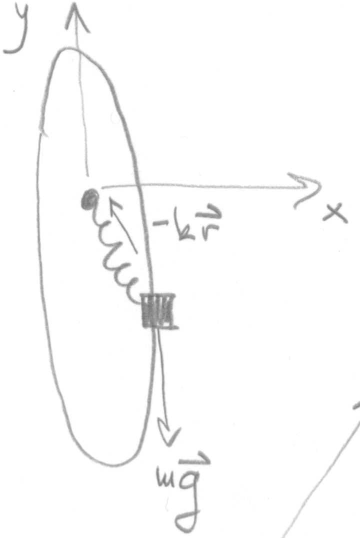
then $v_0^2 = \frac{GM_{sun}}{R_{merc}}$ at $r = R_{merc}$,

$$\Rightarrow v_{max}^2 = \frac{2(2pAn_{merc}R^2_{merc} - GmM_{sun})}{mR_{merc}} + \frac{2mv_0^2}{2m}$$

$$= \frac{4pAn_{merc}R_{merc}}{m} - \frac{2GM_{sun}}{R_{merc}} + \frac{GM_{sun}}{R_{merc}}.$$

$$v_{max} = \sqrt{\frac{4pAn_{merc}R_{merc}}{m} - \frac{GM_{sun}}{R_{merc}}} \;//.$$

③



$$m\vec{a} = \vec{F}_{net} = -k\vec{r} + m\vec{g}$$

$$m a_x = m \frac{d^2x}{dt^2} = -kx$$

$$m a_y = m \frac{d^2y}{dt^2} = -ky - mg$$

$$\boxed{y(t) = A\cos(\omega t) - \frac{mg}{k}}$$

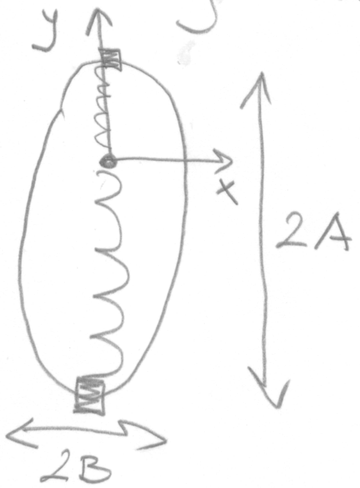
satisfies equation with $\omega = \sqrt{\frac{k}{m}}$

$$\frac{dy}{dt}(t) = -\omega A \sin(\omega t)$$

$$\frac{d^2y}{dt}(t) = -\omega^2 A \cos(\omega t)$$

Similarly $x(t) = -B\sin(\omega t)$

but we don't need this



$R_A = \max y(t)$ (when $\cos = 1$)

$$R_A = A - \frac{mg}{k}$$

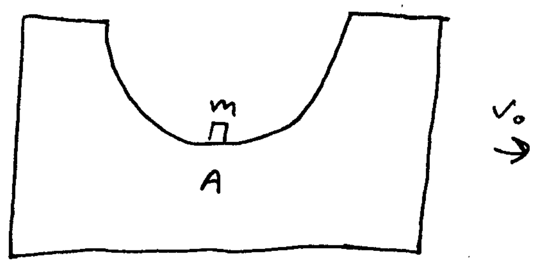
$R_B = |\min y(t)|$ (when $\cos = -1$)

$$R_B = A + \frac{mg}{k}$$

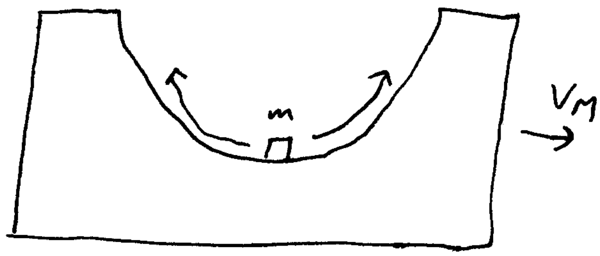
We don't know A or B but we can solve this problem:

$$R_B - R_A = A + \frac{mg}{k} - \left(A - \frac{mg}{k}\right) = \boxed{\frac{2mg}{k}}$$

④

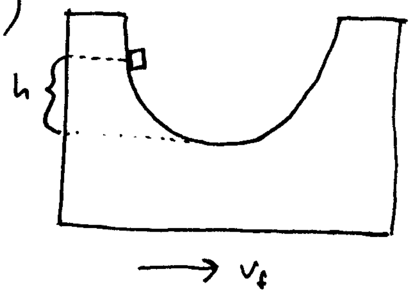


$t=0$



$t>0$

a)



Momentum is conserved, and when the small block reaches its maximum height it is at rest w.r.t. the big block, so both blocks are ~~at~~ moving with velocity v_f.

$$p_0 = p_f$$

$$Mv_0 = (M+m)v_f$$

$$\boxed{v_f = \frac{M}{M+m} v_0}$$

Energy is conserved:

$$\tfrac{1}{2} Mv_0^2 = \tfrac{1}{2}(M+m)v_f^2 + mgh = \frac{1}{2}\frac{M^2 v_0^2}{M+m} + mgh$$

$$h = \frac{v_0^2 M}{2mg}\left(1 - \frac{M}{M+m}\right) = \boxed{\frac{v_0^2}{2g}\left(\frac{M}{M+m}\right)}$$

b) In CM frame, the small mass moves to the left and rises to h, then falls back down and reaches its maximum velocity at A.

Momentum is conserved

$$Mv_0 = mv_m + Mv_M$$

$$v_M = \frac{Mv_0 - mv_m}{M}$$

Energy is conserved

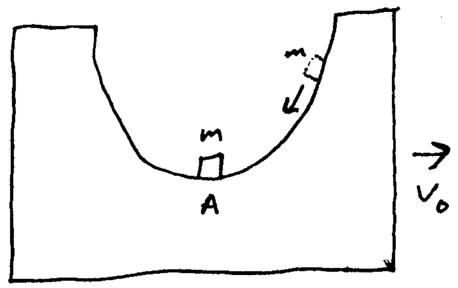
$$\frac{1}{2}Mv_0^2 = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2 = \frac{1}{2}mv_m^2 + \frac{1}{2}M\left(\frac{Mv_0 - mv_m}{M}\right)^2$$

$$\cancel{Mv_0^2} = mv_m^2 + \cancel{Mv_0^2} + \frac{m^2}{M}v_m^2 - 2mv_0v_m$$

$$v_m\left(1+\frac{m}{M}\right) = 2v_0$$

$$\boxed{v_m = \frac{2v_0}{1+\frac{m}{M}}}$$

c)



When m approaches A from the right, the system will be in its initial state. This happens for $\boxed{\text{odd } N}$

Problem 5

Conservation of momentum

$$m_a\vec{v}_0 = m_a\vec{v}_{fa} + m_b\vec{v}_{fb}$$

This is an elastic collision, so we can conserve kinetic energy

$$\frac{1}{2}m_a v_0^2 = \frac{1}{2}m_a v_{fa}^2 + \frac{1}{2}m_b v_{fb}^2$$

For $m_a = m_b$

$$\begin{aligned}
&\vec{v}_0 = \vec{v}_{fa} + \vec{v}_{fb} \text{ and } v_0^2 = v_{fa}^2 + v_{fb}^2 \\
&\vec{v}_0 \cdot \vec{v}_0 = (\vec{v}_{fa} + \vec{v}_{fb}) \cdot (\vec{v}_{fa} + \vec{v}_{fb}) \\
&\Rightarrow v_0^2 = v_{fa}^2 + v_{fb}^2 + 2\vec{v}_{fa} \cdot \vec{v}_{fb} \\
&\Rightarrow v_{fa}^2 + v_{fb}^2 = v_{fa}^2 + v_{fb}^2 + 2\vec{v}_{fa} \cdot \vec{v}_{fb} \\
&\Rightarrow \vec{v}_{fa} \cdot \vec{v}_{fb} = 0
\end{aligned}$$

Since $\vec{v}_{fa} \cdot \vec{v}_{fb} = 0$, the angle between $\vec{v}_{fa}$ and $\vec{v}_{fb}$ is 90° ■