

**ME 40**  
**Thermodynamics**  
**Spring 2009**

**Quiz #3**

**March 16, 2009**

**Name:** \_\_\_\_\_

**SID:** \_\_\_\_\_

Instructions:

Read each question carefully. Take into consideration the point values for each question. **Write your name and SID on each page.** You have roughly 45 minutes.

One double-sided reference sheet is permitted.

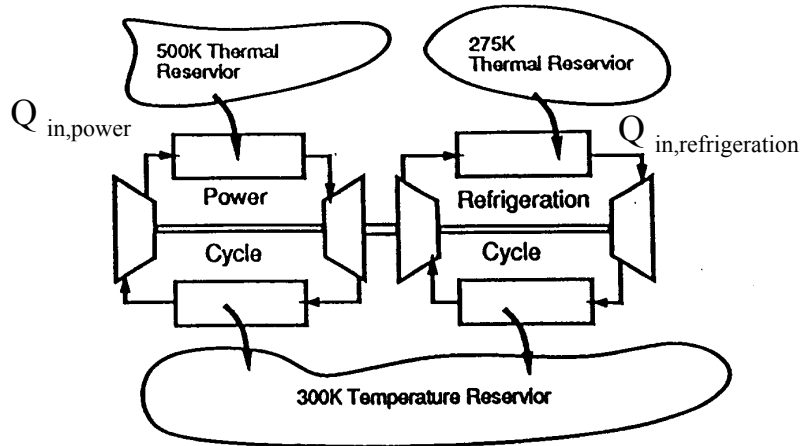
Good luck!

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### Question 1: [10 points]

Consider a device consisting of two cycles as sketched below. The heat engine cycle (the power cycle) operates between two thermal reservoirs at 500 K and 300 K respectively. The refrigeration cycle interacts with reservoirs at 300 K and 275 K respectively. The power from the heat engine cycle is used entirely to run the refrigeration cycle. Both cycles are assumed to be ideal Carnot cycles. Determine the ratio of heat inputs into these two cycles,  $Q_{in,refrigeration}/Q_{in,power}$



solution:

The work from Carnot cycle  $W_{engine} = (1 - 300/500) Q_{in,power} = 0.4 Q_{in,power}$   
 Work required from the Carnot refrigeration cycle =  $Q_{in,refrigeration} / \text{COP}_{Carnot}$   
 where  $\text{COP}_{Carnot} = 1 / (300/275 - 1) = 11$   
 Work required from the Carnot refrigeration cycle =  $Q_{in,refrigeration} / 11$   
 Setting Work required from the Carnot refrigeration cycle =  $W_{engine}$

$$Q_{in,refrigeration} / 11 = 0.4 Q_{in,power}$$

$$Q_{in,refrigeration} / Q_{in,power} = 4.4$$

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## Question 2: [5 points]

An ideal gas initially at 300 K and 500 kPa (State 1) passes through an *adiabtic* throttling device. The pressure of this ideal gas drops to 100 kPa (State 2). Determine 1) the change of entropy per unit mass of this ideal gas (2 points), and 2) the minimum work required to reverse the throttling process from State 2 to State 1 (3 points). The gas constant of this ideal gas is  $R=0.3$  kJ/kg-K.

1<sup>st</sup> law gives  $T_2=T_1$  as  $h_2=h_1$ .

Second law: the entropy change for an ideal gas is

$$\Delta s = C_p \ln(T_2/T_1) - R \ln(P_2/P_1) = -R \ln(P_2/P_1) = -0.3 \text{ kJ/kg-K} \ln(0.2) = 0.48 \text{ kJ/kg-K}$$

The minimum work required is equal to the loss of potential work =  $T \Delta s = 300 \text{ K} * 0.48 \text{ kJ/kg-K} = 144 \text{ kJ/kg}$ . Alternatively, one can compute the reversible work for an isothermal compression process from State 2 to State 1:  $w_{rev} = TR \ln(P_1/P_2) = 144 \text{ kJ/kg}$ .

## Question 3: [5 points]

An inventor claims that a new refrigeration compressor has been designed to receive saturated R-134a vapor at  $-20^\circ\text{C}$  (entropy of this vapor is  $s_g=0.9456$  kJ/kg-K) and delivers superheated vapor at 0.7 MPa,  $30^\circ\text{C}$  (entropy  $s_{super}=0.9313$  kJ/Kg-K). The compression process has a net heat *loss* of 5kJ/kg to the surroundings at 300 K. Perform a steady-flow analysis to determine if the compression process violates the second law?

For steady-flow process, we have

$s_{gen} = s_{out} - s_{in} - q/T = 0.9313 - 0.9456 + 5/300 = 0.002367 \text{ kJ/kg-K} > 0$ ; no violation of second law.

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### Question 4: [10 points]

Steam at 6 MPa and 500°C enters a two-stage *adiabatic* turbine at a rate of 10 kg/s. Ten percent of the steam (mass flow rate = 1 kg/s) is extracted at the end of the first stage at a pressure of 200 kPa. The remainder (mass flow rate = 9 kg/s) is further expanded in the second stage and leaves turbine at 20 kPa. Determine the **maximum possible** power output of this turbine (sum of 1<sup>st</sup> and 2<sup>nd</sup> stages). Note: properties of steam at 6 MPa and 500°C:  $v=0.05667 \text{ m}^3/\text{kg}$ ,  $u=3083.1 \text{ kJ/kg}$ ,  $h=3423.1 \text{ kJ/kg}$ ,  $s=6.8826 \text{ kJ/kg-K}$ .

Solution: exit stage I:  $s=6.8826 = s_f + x s_{fg} = 1.5302 + x * 5.596 \rightarrow x=0.9565$

$$h_1 = 504.71 + x * 2201.6 = 2610.47 \text{ kJ/kg}$$

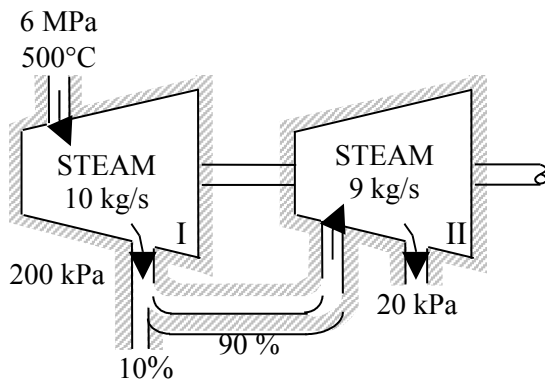
$$\text{power produced in the first stage} = 1 \text{ kg/s} (h_{\text{inlet}} - h_1) \text{ kJ/kg} = 812.63 \text{ kw}$$

second stage:  $s=6.8826 = 0.832 + x * 7.0752 \rightarrow x=0.8552$

$$h_2 = 251.42 + x * 2357.5 = 2267.5 \text{ kJ/kg}$$

$$\text{power from stage II} = 9 \text{ kg/s} (h_{\text{inlet}} - h_2) \text{ kJ/kg} = 9 \times (3423.1 - 2267.5) = 10400.24 \text{ kw}$$

$$\text{total} = 11,212.88 \text{ kw}$$



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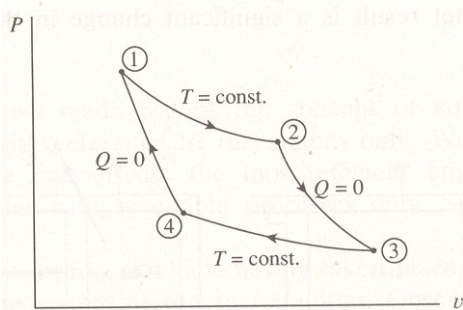
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### Question 5: [10 points]

A Carnot engine operating on **air** accepts 50 kJ/kg of heat from a high temperature reservoir at  $T_H$  and rejects 20 kJ/kg to a low temperature reservoir at  $T_L$  as sketched below. The pressure after isothermal expansion is 200 kPa (State 2) and the specific volume at State 3 is  $10 \text{ m}^3/\text{kg}$ .

- 1) Determine the temperature ratio of high temperature and low temperature reservoirs,  $T_H/T_L$  [2 points].
- 2) Determine the pressure at State 3. [5 points]
- 3) Determine  $T_H$  and  $T_L$ . [3 points]

Note: isentropic (adiabatic & reversible) compression and expansion processes satisfy the relationship  $Pv^k = \text{constant}$ . For air,  $k=1.4$ , and  $R_{\text{air}} = 0.287 \text{ kJ/kg}\cdot\text{K}$  and we will assume ideal gas law for air:  $Pv=R_{\text{air}} T$ .



P-v diagram for Carnot engine

1→2: Isothermal expansion

2→3: Adiabatic & reversible expansion

3→4: Isothermal compression

4→1: Adiabatic & reversible compression

**1)** The temperature reservoir ratio is  $T_H/T_L = 50/20 = 2.5$ .

2) Using the isentropic relation

$$P_3 v_3^k = P_2 v_2^k$$

and the ideal gas law we have  $(T_2/T_3) = (P_2/P_3)^{(k-1)/k}$

Note that the reservoir temperature  $T_H$  is the same as  $T_2$  as this is a reversible heat transfer so that  $(T_2/T_3) = (T_H/T_L) = 2.5$

$$\rightarrow P_3 = 8.1 \text{ kPa}$$

3) Using ideal gas law  $P_3 v_3 = R_{\text{air}} T_3 \rightarrow T_L = T_3 = P_3 v_3 / R_{\text{air}} = (8.1 * 10) / 0.287 = 282 \text{ K}$   
 $T_H = 2.5 * T_L = 705.2 \text{ K}$