University of California at Berkeley
Department of Physics
Physics 8B, Fall 2008 Section 2

Midterm 1
October 7, 2008

You will be given 110 minutes to work this exam. No books are allowed, but you may use a handwritten formulae sheet no larger than one side of an 8 ½" by 11" sheet of paper. Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, and take particular care to explain what you are doing. Please use the symbols described in the problems, tell us why you're writing any new equations, and label any drawings that you make. Write your answers directly on the exam, and if you have to use the back of a sheet make sure to put a note on the front. Do not use a blue book or scratch paper.

NAME: Mike DeWeese

SID NUMBER: __________

DISCUSSION SECTION NUMBER: __________

DISCUSSION SECTION DATE/TIME: __________

1 (25)	
2 (25)	
3 (25)	
4 (25)	
Total (100)	

Formulae: (You may not need all of these!)

$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$ $\vec{E} = k\frac{q_1}{r^2}\hat{r}$ $V = k\frac{q_1}{r}\hat{r}$ $k = \frac{1}{4\pi\varepsilon_0}$ $\vec{\tau} = \vec{p}\times\vec{E}$ $p = qd$

$\varepsilon_0\Phi_E = \varepsilon_0\oint\vec{E}\bullet d\vec{A} = q_{enc}$ $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ $\Delta V = \frac{\Delta U}{q} = -\int_i^f\vec{E}\bullet d\vec{s}$ $a = \frac{v^2}{r}$

$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$ $V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$ $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$ $x = x_0 + v_0 t + \frac{1}{2}at^2$

$q = CV$ $C = \frac{\varepsilon_0 A}{d}$ $C = 2\pi\varepsilon_0\frac{L}{\ln(b/a)}$ $C = 4\pi\varepsilon_0\frac{ab}{b-a}$ $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

$I = \frac{dq}{dt}$ $I = \frac{V}{R}$ $P = IV = I^2R = \frac{V^2}{R}$ $KE = \frac{1}{2}mv^2$

$R_{eq} = \sum R_i$ $R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$ $qvB = \frac{mv^2}{r}$ $\vec{F}_B = I\vec{L}\times\vec{B}$ $\vec{\tau} = \vec{\mu}\times\vec{B}$

$B = \frac{\mu_0 I}{2\pi R}$ $\oint\vec{B}\bullet d\vec{s} = \mu_0 I_{enc}$ $B = \mu_0 In$ $B = \frac{\mu_0 IN}{2\pi}\frac{1}{r}$

$\Phi_B \equiv \int\vec{B}\cdot d\vec{A} = BA$ $\oint\vec{E}\cdot d\vec{s} = emf = -\frac{d\Phi_B}{dt} = -N\frac{d\Phi_B}{dt}$

$L \equiv \frac{N\Phi_B}{I}$ $L = \mu_0 n^2 Al$ $V = L\frac{dI}{dt}$ $U = \frac{1}{2}LI^2$

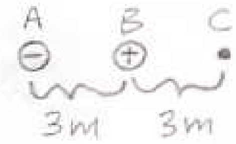
$V(t) = L\frac{dI(t)}{dt}$ $I(t) = \frac{1}{L}\int V(t)dt$ $V(t) = \frac{1}{C}\int I(t)dt$ $I(t) = C\frac{dV(t)}{dt}$

$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ N\cdot m^2/C^2$

$\varepsilon_0 = 9\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$ charge on an electron $= -1.6\times10^{-19}\,C$

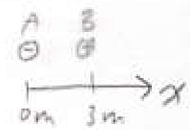
$\mu_0 = 1\times10^{-6}\ \mathrm{H/m}$

Problem 1 – Point charges and potential energy.
Two point charges are held in place 3m apart, as shown in the diagram. The left charge has $Q_A = -2\times10^{-9}$C and the right one has $Q_B = +4\times10^{-9}$C.



a) What is the magnitude and direction of the force on charge B due to charge A?
b) What is the electric field strength and direction at point C, which is 3m to the right of point B?
c) What is the electric potential at point C? Please use the convention that V = 0 very far away from the charges.
d) Now a third point charge with $Q_C = +3\times10^{-9}$C is brought in to point C from very far away while charges A and B are held in place. How much work does this require? Take care with signs.
e) What is the total potential energy stored in the final configuration of all three charges? Please use the convention that PE = 0 if all three charges are separated far from each other.

a) $\vec{F}_{onB} = k\frac{q_A q_B}{r_{A\to B}^2}\hat{r}_{A\to B} = 9\times10^9\frac{Nm^2}{C^2}\frac{(-2\times10^{-9}C)(4\times10^{-9}C)}{(3m)^2}\hat{i}$



$= -\frac{\cancel{9}\cdot2\cdot4\times10^{\cancel{9}-\cancel{9}-9}}{\cancel{9}}N\hat{i}$

$= \boxed{-8\times10^{-9}N\hat{i}}$ to the left

b) $\vec{E}_c = k\frac{q_A}{r_{A\to c}^2}\hat{r}_{A\to c} + k\frac{q_B}{r_{B\to c}^2}\hat{r}_{B\to c} = k\left(\frac{q_A}{r_{A\to c}^2}+\frac{q_B}{r_{B\to c}^2}\right)\hat{i}$

$= 9\cdot10^9\frac{Nm^2}{C^2}\left(\frac{-2\times10^{-9}C}{(6m)^2}+\frac{4\times10^{-9}C}{(3m)^2}\right)\hat{i}$

$= \left(-\frac{2\times9}{36}+4\right)\frac{N}{C}\hat{i}$

$= \left(-\frac{1}{2}+4\right)\frac{N}{C}\hat{i} = \boxed{3.5\frac{N}{C}\hat{i}}$ to the right

c) point charge: $V = k\frac{q_{source}}{r}$ (one can get this by: $V = \int_\infty^r d\vec{r}\cdot\vec{E} = \int_\infty^r dr\frac{kq}{r^2} = \frac{kq}{r}\Big|_\infty^r = \frac{kq}{r}-\frac{kq}{\infty} = \frac{kq}{r}$)

$V_{at\,c} = \sum_i^{sources} k\frac{q_i}{r_i} = k\frac{q_A}{r_{A\to c}} + k\frac{q_B}{r_{B\to c}}$

$= 9\times10^9\frac{Nm^2}{C^2}\left(\frac{-2\times10^{-9}C}{6m}+\frac{4\times10^{-9}C}{3m}\right)$

$= (-3+12)\frac{N}{m\cdot C}$

$= \boxed{9\,V}$

d) Work done on charge C $= \Delta PE = PE_f - \cancel{PE_i}^{\to 0}$ $PE_i = 0$ since $PE(r=\infty) = 0$

$PE_f = PE_{at\,c} = q_c\cdot V_c = (+3\times10^{-9}C)\cdot 9V = \boxed{2.7\times10^{-8}J}$ work on q_c is positive

(please see back)

e) $PE_{total} = \sum_{\substack{\text{all} \\ \text{pairs}}} k \frac{q_i q_j}{r_{i\,to\,j}}$

$$= k\frac{q_A q_B}{r_{AB}} + k\frac{q_B q_C}{r_{BC}} + k\frac{q_C q_A}{r_{AC}}$$

$$= 9\times10^{9}\,\frac{Nm^2}{C^2}\left(\frac{(-2\times10^{-9}C)(+4\times10^{-9}C)}{3m} + \frac{(4\times10^{-9}C)(3\times10^{-9}C)}{3m} + \frac{(-2\times10^{-9}C)(3\times10^{-9}C)}{6m}\right)$$

$$= \left(\frac{-9\cdot2\cdot4}{3} + \frac{9\cdot4\cdot3}{3} - \frac{9\cdot2\cdot3}{6}\right)\times10^{+9-9-9}\ \frac{Nm}{C}$$

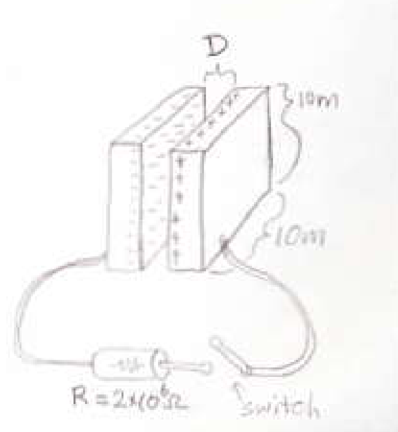
$$= (-24+36-9)\times10^{-9}\ J$$

$$= (-24+27)\times10^{-9}\ J$$

$$= \boxed{+3\times10^{-9}\ J}$$

Problem 2 – Big Capacitor

Two flat square metal plates that are each 10m on a side are separated by a distance $D = 3\times10^{-4}$m. The left plate has a total charge of $Q_L = -6C$, while the right plate is charged up to $Q_R = +6C$. A wire connects the left plate to a resistor with resistance $R = 2\times10^{6}\Omega$, but the right plate is initially disconnected from the resistor due to an open switch.



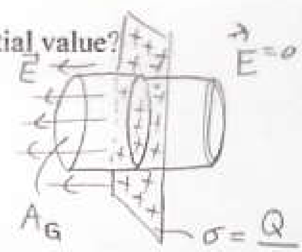
a) What is the electric potential difference between the plates? Is the potential higher on the left or right plate?

b) What is the strength and direction of the electric field between the plates?

c) If a small charged piece of plastic with mass $m = 2\times10^{-8}$kg is released from rest at the left plate, and it accelerates until it achieves a final speed of 20 m/s at the right plate, then what is the sign and magnitude of the charge? You may neglect gravity.

d) *Immediately* after closing the switch, what is the current through the resistor?

e) How long after the switch is closed will the current drop to $1/e^3 \approx 0.05$ of its initial value?



a) Area $= A = (10\,m)(10\,m) = 100\,m^2$

parallel plate capacitor:

$$V = D\cdot E = \frac{D\cdot\sigma}{\varepsilon_0} = \frac{D\cdot Q}{A\,\varepsilon_0}$$

Gauss's Law: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$$E A_G = \frac{A_G\,\sigma}{\varepsilon_0}$$

$$\sigma = \frac{Q}{A_{plate}} = \frac{6C}{100m^2}$$

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow V = D\cdot E = \frac{D\cdot\sigma}{\varepsilon_0} = \frac{D\cdot Q}{A_{plate}\,\varepsilon_0}$$

uniform $\vec{E}$

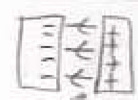
$\vec{E}$ parallel w/ path

$$V = \frac{3\times10^{-4}m\cdot 6C}{100\,m^2\cdot 9\times10^{-12}\frac{C^2}{Nm^2}}$$

$$= 2\times10^{6}\,\frac{N}{C} = \boxed{2\times10^{6}\,V}$$

potential is higher on positively charged plate which is the Right plate

b) $$|\vec{E}| = E = \frac{V}{D} = \frac{2\times10^{6}V}{3\times10^{-4}m} = \boxed{0.7\times10^{10}\,\frac{V}{m}}$$ to the Left



c) from left to right $\Rightarrow \vec{F}_E$ on plastic particle goes against $\vec{E}$ so charge must be negative

Cons. of Energy: $[PE_f + KE_f] = [PE_i + \cancel{KE_i}^{\,0}]$

$$KE_f = PE_i - PE_f$$

$$\frac{1}{2}m v_f^2 = q(V_i - V_f)$$

$$q = -\frac{1}{2}m v_f^2 / V$$

$$= -\frac{1}{2}\,2\times10^{-8}kg\,(20\tfrac{m}{s})^2 / 2\times10^6 V$$

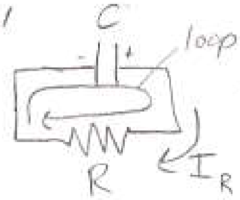
$$= -\frac{400}{2}\times10^{-8-6}\,\frac{J}{V}$$

$$= \boxed{-2\times10^{-12}\,C}$$

(please see back)

d) immediately after switch is closed, capacitor still has nearly all $Q = 6C$ of charge on each plate, so Voltage across resistor is the same as the voltage across capacitor before switch was closed:

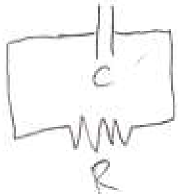
Kirchhoff's law for Voltages around a loop:



$$+V_c - I_R R = 0$$

$$I_R = \frac{V_c}{R} = \frac{2\times10^6\,V}{2\times10^6\,\Omega} = \boxed{1\ A}$$

e) RC circuit



$$\tau = RC$$

$$C = \frac{Q}{V} = \frac{6C}{2\times10^6 V} = 3\times10^{-6}\,F = 3\,\mu F$$

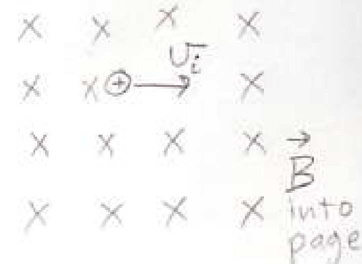
$$\tau = RC = (2\times10^6\,\Omega)(3\times10^{-6}\,F) = 6\,s$$

$$I(t) = I(t{=}\infty) + I(t{=}0)\cdot e^{-t/\tau}$$
$$= 0 + I_0\, e^{-t/RC}$$

set this to $I_0\, e^{-3}$:

$$I_0\, e^{-t/RC} = I_0\, e^{-3}$$
$$e^{-t/RC} = e^{-3}$$
$$\frac{-t}{RC} = -3$$
$$\frac{t}{RC} = 3$$
$$t = 3RC$$
$$= (3)(6\,s)$$
$$= \boxed{18\,s}$$

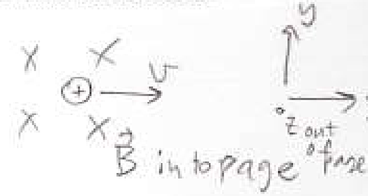
Problem 3 – Charged Particle in Magnetic Field
At t = 0, an exotic particle with charge Q = +1C and mass m = 1g is moving to the right with an initial speed of v_i = 10m/s. There is a uniform magnetic field of 2T into the page. You may neglect gravity for this problem.



a) What is the initial direction and magnitude of the magnetic force on the moving particle?
b) What is the radius of the circular path followed by the particle?
c) How much work is done on the particle by the magnetic field during ¼ of a revolution about its circular path?
d) How long will it take for the particle to make one full revolution?
e) If a second particle with charge Q = -3C is traveling straight out of the page, what is the force on that particle due to the uniform magnetic field?

a) $\vec{F}_B = q\, \vec{v}_i \times \vec{B} = +1C\ 10\frac{m}{s}\ 2T\ \cancel{\sin(\theta)}^{\,1}\ \hat{j}$

$= \boxed{20N\ \hat{j}}$ upwards in diagram



b) for uniform circular motion: $a = \frac{v^2}{r}$ N2L: $\vec{F}_{net} = m\vec{a}$

so: $r = \frac{v^2}{a} = \frac{v^2}{F_{net}/m}$ neglect gravity $\Rightarrow \vec{F}_{net} = \vec{F}_B$ ✓

$= \frac{mv^2}{F_B}$

$= \frac{0.001\ kg\ (10\frac{m}{s})^2}{20N}$

$= \frac{1}{2} \times 10^{-3+2-1}\ m$

$= \boxed{5 \times 10^{-3}\ m}$ = 5mm

c) $W_{on} = \int_A^B d\vec{r} \cdot \vec{F}_{on}$

$W_{\substack{on\ q \\ by\ \vec{B}}} = \int_{1/4\ circle} d\vec{r} \cdot \vec{F}_B = \frac{1}{4}(2\pi r) \cdot F_B \cancel{\cos(\tfrac{\pi}{2})}^{\,0} = \boxed{0}$

$\vec{B}$ does no work on moving charges

$\vec{B}$ alway orthogonal to $d\vec{r}$

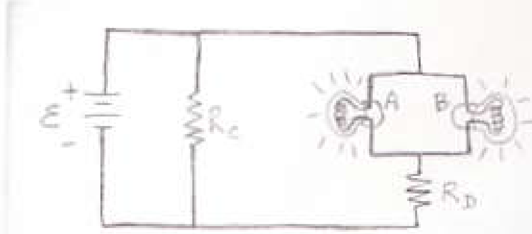
d) full period $= T = \frac{(\text{path length})}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi\ 5\times10^{-3} m}{10\ m/s} = \pi \times 10^{-3} s$

$\cong \boxed{3\ ms}$

e) $\vec{F}_B = q\,\vec{v} \times \vec{B} = q v B \cancel{\sin(0)}^{\,0} = \boxed{0}$ because $\vec{B}$ is parallel to $\vec{v}$

Problem 4 – Simple Circuit

Consider the circuit diagram at the right, which contains two lightbulbs, labeled A and B, each with a resistance of 80Ω, and two resistors with $R_C = R_D = 20\Omega$, and a battery with an emf = 30V.



a) What is the voltage across R_C?

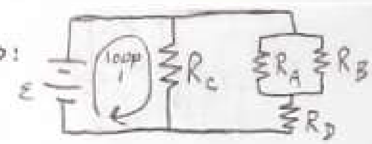
b) How much current is passing through R_C? Is the (positively defined) current passing through R_C going upwards or downwards as viewed in the diagram?

c) What is the voltage across R_D?

d) How much power is being dissipated in R_D?

e) If one of the two lightbulbs burns out so that its filament is broken, how much power will be dissipated in R_D from then on?

a) Kirchoff's sum of voltages rule around a closed loop:



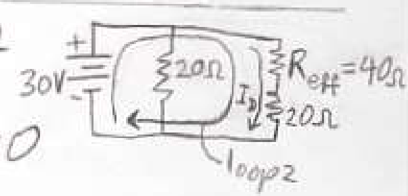
$$\sum_{\text{loop 1}} V = +\mathcal{E} - V_c = 0 \Rightarrow V_c = \mathcal{E} = \boxed{30V}$$

V_c = voltage drop across R_c

b) $V_c = I_c R_c$ $\quad I_c = \frac{V_c}{R_c} = \frac{30V}{20\Omega} = \boxed{1.5A}$ downward since voltage is higher at the top of resistor

c) Light bulbs are in parallel, so: $R_{eff} = \frac{R_A R_B}{R_A + R_B} = \frac{80 \cdot 80\Omega^2}{(80+80)\Omega} = 40\Omega$

Kirchhoff's Voltage sum rule:



I_D is current passing through R_{eff} and R_D

$$\sum_{\text{loop 2}} V = +\mathcal{E} - I_D R_{eff} - I_D R_D = 0$$

$$+30V - I_D 40\Omega - I_D 20\Omega = 0$$

$$I_D = \frac{30V}{60\Omega} = \frac{1}{2}A$$

$$V_D = I_D R_D = \frac{1}{2}A \cdot 20\Omega = \boxed{10V}$$

d) $P_D = I_D V_D = \frac{1}{2}A \cdot 10V = \boxed{5W}$

e)

Kirchhoff's loop rule: $\sum_{\text{loop 2}} V = +\mathcal{E} - 80\Omega \cdot I_D - 20\Omega \cdot I_D = 0$

$$I_D = \frac{\mathcal{E}}{80\Omega + 20\Omega} = \frac{30V}{100\Omega} = 0.3A$$

$$P_D = I_D V_D = I_D^2 R_D = (0.3A)^2 20\Omega = 0.09A^2 \cdot 20\Omega = \boxed{1.8W}$$